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AN

INTRODUCTION
TO
Natural Philoſophy:
OR,
Philoſophical Lectures
Read in the
University of *Oxford,*
Anno Dom. 1700.

To which are Added,
The DEMONSTRATIONS of Monſieur HUYGENS's *Theorems*, concerning the Centrifugal Force and Circular Motion.

By JOHN KEILL M. D.
Savilian Profeſſor of Aſtronomy. *F. R. S.*

Tranſlated from the laſt Edition of the Latin.

THE FOURTH EDITION.

LONDON:
Printed for M. SENEX, W. INNYS, T. LONGMAN and T. SHEWELL. MDCCXLV.

1860 e. 2

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THE PREFACE.

ALTHOUGH now-a-days the Mechanical Philoſophy is in great Repute, and in this Age has met with many who cultivate it; yet in moſt of the Writings of the Philoſophers, there is ſcarce any thing Mechanical to be found beſides the Name. Inſtead whereof, the Philoſophers ſubſtitute the Figures, Ways, Pores, and Interſtices of Corpuſcles, which they never ſaw; the inteſtine Motion of Particles, the Collectations and Conflicts of Acids and Alkalies, and the Events that thence ariſe, they relate ſo exactly, that there is nothing but a Belief wanting in the Hiſtory of Nature, as often as they ſet forth the Miracles of their ſubtile Matter: I ſay, Miracles, for certainly that muſt be a ſort of a Miracle, which happens contrary to the well-known Laws of Nature, and the eſtabliſhed Principles of

Mechanicks; *as would be all the Phenomena of Nature, if they were produced by a ſubtile Matter, and the Method of Operation that is delivered by the Philoſophers.*

TO *explain the Works of Nature, they make uſe of ſuch* Poſtulata, *as can neither be conceded, or underſtood*; *and which are indeed more complicated than the Phenomena themſelves, whoſe Cauſes they inveſtigate. Beſides, if their* Poſtulata *are granted, yet the Effects cannot be thence deduced, whoſe Cauſes and Origin they boaſt to have diſcovered.*

BUT *that nobody may ſuſpect that this is raſhly and malevolently aſſerted by us, we ſhall here examine the Theory they have framed to explain the moſt univerſal Affection of all Terreſtrial Bodies*; *I mean Gravity, which they principally boaſt of having been able to deduce by Mechanical Laws from the Action of a ſubtile Matter.*

THE Carteſians *will have Gravity to ariſe from the Action of a Celeſtial Matter, which being agitated in a Vortex, is carried about the Earth, and conſequently recedes, as much as it can, from the Earth, and forces Terreſtrial Bodies being leſs agitated towards the Earth. Or, as the Moderns have more clearly explained the Opinion of* Cartes, *ſince the ethereal Matter makes continual Gyrations about the Earth, it will have, after the manner of Bodies that move circularly, an Endeavour of receding from the Center of Motion, ſo that the Terreſtrial Bodies having a leſs Force, will be compelled by it towards the Center*; *as Water gravitating towards the Earth, drives upwards or towards the Circumference, Bodies of a leſs Weight in reſpect to their Bulk, which are immerſed in it.*

THIS, *however ſpecious it may ſeem at firſt appearance, if it be brought to an Examination, will be found contrary to almoſt all the Laws of Nature. For in the firſt place, the* Carteſians *require their Ethereal Matter to be carried round the Earth in Circles; but from what Cauſe that Motion ariſes, or by what Means it is continued, is as difficult to explain, as to give a Reaſon for Gravity itſelf. They therefore who contend that Gravity proceeds from thence, undertake to explain what is unknown by what is ſtill more unknown, eſpecially ſince there may be produced divers Arguments, whereby that Rotation is wholly deſtroyed. But let us grant to the* Carteſians *their Poſtulatum, and then let us ſee, whether this Phenomenon does thence follow. Since it is neceſſary that the Velocity of the Vortex revolving about the Earth, is, at the Superficies of the Earth, equal to the Velocity of the Earth's Rotation, (for if it was greater, ſome part of the Motion would be communicated to the Earth; whence it would happen, that its Velocity would be always diminiſhed, and that of the Earth increaſed, till they arrived at an Equality) ſo that from the Magnitude of the Earth, and the Time of its Rotation being known, there will be given the Space, which a Body, urged by the Centrifugal Force of the Celeſtial Matter, can run over in a given Time; namely, that which is equal to the Square of the Arch deſcribed in the mean while, applied to the Diameter of the Circle: By* Lemma 2. *in the Demonſtration of Monſ.* Huygens'*s Theorems of the Centrifugal Force and Circular Motion. If a Calculation be made from this Principle, it will be found that the Space which will be run over in a Second of Time, by a Body agitated by a Centrifugal Force of the Æther, will not ex-*

ceed half a Foot: If therefore the Effect of Gravity was produced mechanically, heavy Bodies in a Second of Time would not deſcend above half a Foot; but heavy Bodies in their Motion downwards will paſs over in the ſame Time fifteen Feet. So that if after this manner the Æther was the Cauſe of Gravity, it would act contrary to the Laws of Mechanicks, by making a Body deſcend through fifteen Feet in one Second of Time.

TO *avoid the Force of this Objection, the* Carteſians *ſuppoſe the Rotation of the ethereal Matter to be much ſwifter than the Rotation of the Earth. Which, though it is impoſſible, yet if we alſo concede to them this, there will not thence proceed the Mechanical Action of Gravity. For ſince the Matter of the Vortex is always carried in Circles parallel to the Equator, and the Directions of the Centrifugal Forces are always made in Lines lying in the Planes of theſe Circles; it follows, that all Bodies muſt deſcend in theſe Planes, and tend perpendicularly to the Axis, and not to the Surface of the Earth. If therefore the ſubtile Matter acted mechanically, it would force Bodies perpendicularly to the Axis; whence ſince, according to theſe Theoriſts, it compels them to tend to the Center of the Earth, it produces an Effect contrary to the true Laws of Mechanicks.*

THAT *they may remove this Difficulty, they farther ſuppoſe the ethereal Matter to be carried not in Circles parallel to the Equator, but in great Circles of a Sphere. But how it is poſſible to conceive this, I am at a loſs to know; for ſince every great Circle twice interſects all the other Circles that are infinite in number, it is neceſſary that the Motion of any Particle ſhould be hindered by infinite* others moving different ways, and at length its Motion ſhould ceaſe, if at firſt an equal Quantity of Motion was impreſſed on all the Parts; or that, laſtly, it ſhould be all carried in parallel Circles, if at the beginning the Motion was greater towards one Part than the other. Moreover, it may be aſked, whence it comes to paſs that the ethereal Matter does move in the Superficies of the outermoſt Sphere, ſince it has a Centrifugal Force, it ſeems as if it ſhould recede from thence; what then is it that hinders it from doing ſo? They are wont to anſwer, That the ambient Bodies compreſs the Matter in the exterior Sphere, and prevent its eſcaping. But ſince this Matter muſt preſs other Bodies encompaſſing it, it is neceſſary that it communicates a Motion to them, and theſe Bodies in like manner will impreſs a Motion on others encompaſſing them, and ſo the Motion of the ſubtile Matter will be propagated *in infinitum*; whence of neceſſity its Celerity muſt languiſh by degrees.

THERE are many other Difficulties which accompany theſe Mechanical Explications of Gravity; one whereof we ſhall here propoſe, and which extends to all the Theories of this ſort. And it is this: If a Body is after any manner driven downwards by a ſubtile Matter, the Force whereby it is driven, will be neceſſarily as the Number of Particles, by the acting of which together, the Body is conſtrained to tend towards the Earth: but the Number of Particles is as the Superficies of the Body; wherefore the Force whereby the Body is preſſed downwards, will be as its Superficies, and not as the Quantity of Matter in the Body, which is contrary to Experience. Nor will all the reſt of the Hypotheſes, which they frame to account for the Appearance of *other things, be found leſs repugnant to the Laws of Nature, if they are brought to an Examination.*

ALL *theſe Errors ſeem to ſpring from hence; that Men ignorant of Geometry preſume to philoſophize, and to give the Cauſes of Natural Things. For what can we expect but Miſtakes, from ſuch, as having neglected Geometry, the Foundation of all Philoſophy, and being unacquainted with the Forces of Nature, which can only be eſtimated by the means of Geometry, do yet attempt to explain its Operations, by a Method not at all agreeing with the Rules of Mechanicks?*

AMONGST *the Philoſophers of this ſort,* Cartes *leads the Van; who though he was a famous Geometer, yet that he might accommodate himſelf to the idle and common Herd of Philoſophers, made no uſe of Geometry in his Philoſophy; and although he pretended to explain all things mechanically by Matter and Motion, yet he introduced a Philoſophy which was as contrary to the true Laws of Mechanicks, as was poſſible. Thoſe may be reckoned of his Sect, who refuſe the Labour of Philoſophizing aright, that is, Geometrically: And ſuch are by much the greateſt number, being ſcattered almoſt over the whole Earth.*

BUT *although ſo great a part of the Philoſophers have embraced the Shadow of Philoſophy, and not its Subſtance; yet there have not been wanting (nor, I hope, ever will) ſuch as have employed real Pains in diſcovering the true Laws of Nature, and in inveſtigating from thence the Cauſes of Things by Mechanical Principles.*

AMONGST *the antient Philoſophers, the Divine* Archimedes *is of the greateſt Eminence; who, beſides his Geometrical Labours, has left us the Principles of Mechanicks*

nicks and Staticks demonſtrated in two Books de Æquiponderantibus & de Humido Inſidentibus. *After him through a long Series of Years the Mechanical Philoſophy lay in Obſcurity, nor was it cultivated except by a very few Perſons of a more refined Underſtanding. Amongſt whom we may juſtly reckon* Roger Bacon *of* Oxford, *and* Cardan. *But at length, about the beginning of the laſt Age, that noble* Lyncean *Philoſopher* Galileo, *having by the means of Geometry penetrated into the Secrets of Nature, framed a new Science of Motion, and ſhewed a Method whereby the Mechanical Cauſes of Things might be diſcovered. Then the Famous* Torricelli *and* Paſchal *treading in his very Footſteps, improved Philoſophy by new Speculations. And laſtly, the Societies of* London *and* Paris *were inſtituted by two powerful Monarchs, for the Advancement of Philoſophy; whereby the Knowledge of Nature has been increaſed with wonderful Inventions, not only ſuch as terminate in bare Speculation, but with many others which are of common Uſe to Men. It would be a difficult Taſk to reckon up the innumerable Benefits that accrue to Mankind from the Labours of both Societies: nor is it an eaſy matter to ſhew, how much ſucceeding Ages will be obliged to the Geometrical Demonſtrations of the Illuſtrious Monſieur* Huygens, *concerning the Motion of Pendulums*; *or to the curious Experiments of the Honourable Mr.* Boyle, *whereby he has diſcloſed many wonderful Secrets of Nature. Our late Poſterity will peruſe with a grateful Mind* Dr. Wallis'*s Treatiſe of Motion*; *a Work the moſt perfect of its kind. The Philoſophers will now no longer be troubled to find out the Cauſes of Rivers and Winds, ſince the* *they have been delivered by that acute Geometer Dr.* Halley, *tho' before him they have been ſought after in vain.*

ISHOULD *proceed in enumerating the Merits of others towards the real Philoſophy, if I did not find myſelf obliged to ſtop, to mention the great Inventions of Sir* Iſaac Newton, *whoſe prodigious Genius has laid open more and abſtruſer Myſteries of Nature, than Men could ever have hoped for: but ſince it is impoſſible to comprehend his Diſcoveries within the narrow Limits of this Preface, we ſhall only undertake to ſay thus much, That what all our Predeceſſors from Time immemorial have handed down to us concerning the Mechanical Philoſophy, does not amount to the tenth part of thoſe Things, which Sir* Iſaac Newton *alone, through his vaſt Skill in Geometry, has found out by his own Sagacity. But how the Mechanical Principles may be eaſily applied to explain the Affections of ſuch Bodies as are at a diſtance from us, namely, the Motions and Inequalities of the Planets; the learned World has been lately informed by the Philoſophical and Geometrical Elements of Aſtronomy publiſhed by Dr.* Gregory, *the* Savilian *Profeſſor of Aſtronomy: A Work that will laſt as long as the Sun and Moon endure.*

SINCE *therefore ſuch is the State of the Mechanical Philoſophy, that there is no Admittance to it but by the means of Geometry; ſome of my Friends have requeſted of me, that I would undertake to explain to the Youth of this Univerſity its eaſieſt Principles, and ſuch as only depend on the firſt Elements of Geometry; which thing was alſo aſked of me in the moſt obliging manner in the world, by the*

Gentleman who ſubſtituted me in the publick Schools for the undertaking this Work. And this was Sir Thomas Millington *M. D.* Sedleian *Profeſſor of Natural Philoſophy in this Univerſity, and Preſident of the College of Phyſicians in* London; *a Perſon adorned with Learning of every kind. By his Advice it was, that I read the following Lectures in this Univerſity: And in them my principal Care was, that Beginners might have clear and diſtinct Conceptions of the general Affections of Bodies; for all the Errors in Philoſophical Matters have their Origin from the obſcure and falſe Ideas of Things: ſo that I have endeavoured, as much as I was able, to expound clearly the Extenſion, Solidity, and Diviſibility of Body, which have been delivered by others very obſcurely. And then I have explained the Nature and Properties of Motion, which may be eaſily conceived by all unleſs ſome ſort of Philoſophers; and I have thence deduced the Laws of Nature, and have ſhewn that the Force of Gravity, or the Weight of Bodies, is proportionable to the Quantity of Matter in thoſe Bodies, and the Principle whereby, thro' the means of Engines, great Weights are raiſed. Next, I have made manifeſt the Laws of Motion, and the Cauſe of the Acceleration of heavy Bodies depending on them; and by what Proportion the Spaces paſſed over by heavy Bodies, increaſe or decreaſe according to the different Intervals of the Times. To theſe ſucceed the Rules of Congreſs, as well in hard as elaſtick Bodies, and the Manner whereby the Magnitude of a Stroke may be eſtimated: to which I have adjoined the Compoſitions and Reſolutions of Motions, and ſome other Theorems, of no ſmall uſe in Philoſophy. And that the Philoſophers may farther perceive, how far even the*

the Uſe of the Elementary Geometry extends itſelf in the Knowledge of Natural Things, I have demonſtrated from the Elements the very beautiful Theorems of Monſ. Huygens *concerning the Centrifugal Force and Circular Motion.*

LECTURE I.

Of the Method of Philoſophizing.

AS it is our Purpoſe in theſe Lectures to endeavour to deliver the Explication of the Properties and Affections of Bodies, we have thought it not improper, in the firſt place, to ſay ſomething concerning the Sects, and Principles, and Methods of the Philoſophers; and at the ſame time, to declare our own Manner of Proceeding, in the inveſtigating the Cauſes of Natural Things.

AMONGST the various Sects of Philoſophers that have wrote on Phyſical Subjects, there ſeems to have been four of the greateſt Eminence. The firſt were thoſe who attempted to illuſtrate, (ſhall I ſay?) or rather conceal, the Natures of Things, by the Properties of Numbers and Geometrical Figures. Such were the *Pythagoreans* and the *Platoniſts*; for theſe not being willing that their Opinions ſhould be expos'd to the common People, did therefore, as it were, caſt a Vail over them, by the means of Images and

Hieroglyphicks, that they borrowed from Geometry and Arithmetick: nor indeed did they admit their own Diſciples to a thorough Acquaintance with their real Opinions and the greateſt Depths of their Philoſophy, until they had firſt undergone many Years of Probation and Trial. Tho' this Method ſeems extremely well calculated to keep up the Dignity of their Profeſſion, yet it had a very ill effect in reſpect to us, the Succeſſors of theſe Philoſophers. For by this means their Opinions have been handed down to us with ſo much Obſcurity and Darkneſs, that it is almoſt impoſſible to diſcern, thro' the Diſguiſes that they wrapped them up in, what were their true Sentiments concerning the Natures of Things. But however, notwithſtanding the Philoſophy of this Sect appears very myſterious to us, yet we may be certain, from their Manner of delivering it, that they looked upon Geometry and Arithmetick to be abſolutely neceſſary to ſolve the Phenomena of Nature, and that they uſed them to that very purpoſe.

THE ſecond ſort were the *Peripateticks*. This Sect explained their Philoſophy by Matter and Forms, Privations, Elementary Virtues, occult Qualities, Sympathies and Antipathies, Faculties, Attractions, and the like. But theſe Philoſophers, in my Opinion, ſeem not ſo much to have diſcovered the Cauſes of Things, as to have given proper Names to the Things themſelves, and to have invented ſuch Terms, as are very fit to expreſs natural Actions.

THE third Order of Philoſophers are ſuch as proceed upon *Experiments:* and theſe make it their ſole Buſineſs, that the Properties and Actions of all Bodies may be manifeſted to us, by the means of our Senſes. And indeed Philoſophy has received very conſiderable Advantages from their Labours; and it might thereby have been ſtill further improved, if the

Experimenters themſelves had not too often diſtorted their Experiments and Obſervations, in order to favour ſome darling Theories they had eſpouſed.

THE laſt ſort of Philoſophers are ſuch as are wont to be called *Mechanical.* They that liſt themſelves under this Banner, imagine they can explain all the Phenomena of Nature by Matter and Motion, by the Figure and Texture of the Parts, by ſubtle Particles, and the Actions of Effluvia; and they likewiſe contend, that theſe Operations are brought about by the known and eſtabliſhed Laws of Mechanicks.

AMONGST theſe various Ways of Philoſophizing, as there is no particular one, wherein we do intirely acquieſce; ſo in each, there are ſome things which we can approve of. Wherefore we ſhall chuſe out of all of them what may be thought uſeful, and thence compoſe the Method we ſhall here follow.

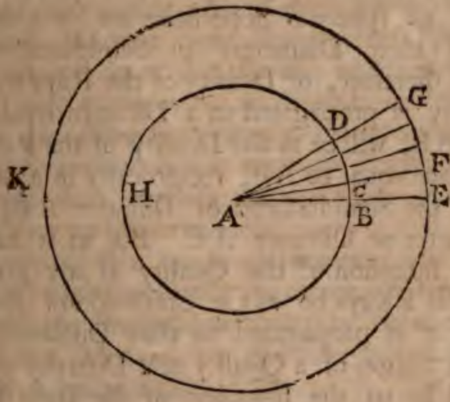
AND firſt of all, in imitation of the ancient *Pythagoreans* and *Platoniſts*, we ſhall call in to our aſſiſtance Arithmetick and Geometry, as Arts very neceſſary to Philoſophy, and without which, but little of Certainty can be ever diſcovered in natural Cauſes. For ſince every Phyſical Action depends upon Motion, or at leaſt it cannot be performed without Motion; the Quantity and Proportion of Motion, the Magnitude, Figures, Number, and Colliſions of Bodies in motion, and their Forces neceſſary to move other Bodies, ought to be inveſtigated. But it is impoſſible to determine all theſe, without being inſtructed in the Nature of Quantity and Proportion: and therefore there is occaſion for thoſe Arts, that demonſtrate their Properties; and conſequently Geometry and Arithmetick muſt be thought neceſſary to any one, that would make any progreſs in Natural Philoſophy.

SECONDLY, we ſhall not be aſhamed to uſe, with the *Peripateticks*, the Terms *Quality*, *Faculty*, *Attraction*, and the like; not that by theſe words we pretend to define the true and phyſical Cauſe and Modus of Action, but as theſe Actions may be augmented and diminiſhed, and therefore ſince they have the Properties of Qualities, the ſame Name may not unfitly be applied to them, ſo that we thereby only mean to expreſs the Ratios of the Forces or their Augmentation and Diminution. For example, we may ſay that Gravity is a Quality, whereby all Bodies are carried downwards, whether its Cauſe ariſes from the Virtue of the central Body, or is innate to Matter itſelf; or whether it proceeds from the Action of the Æther agitated by a centrifugal Force, and ſo tending upwards; or laſtly, whether it is produced after any other manner whatever. So likewiſe we may call the Endeavour of Bodies to approach one another, Attraction; by which word we do not mean to determine the Cauſe of that Action, whether it proceeds from the Action of the Bodies tending mutually to one another, or from their being agitated by Effluvia emitted, or from the Action of the Æther, or Air, or any Medium whatever that impels the Bodies, which float in it, towards one another: ſuch Actions, I ſay, we may expreſs by theſe Words. And if the true Cauſes are hid from us, why may we not call them occult Qualities? Certainly by the ſame right, as in Algebraical Equations we denote the unknown Quantities by the Letter x or y; and not by a very unlike Method we may inveſtigate the Intenſions and Remiſſions of theſe Qualities, which follow from ſome certain ſuppoſed Conditions. We will illuſtrate this by an Example.

HOWEVER ignorant we are of the Nature of Qualities, and how much ſoever the Modus of Operations is concealed from us, we are able notwithſtanding

ſtanding to demonſtrate the following Theorem concerning their Intenſion and Remiſſion. Every Quality or Virtue that is propagated every way in right Lines from a Center, is diminiſhed in a duplicate Proportion of the Diſtance from that Center.

LET A be a Point or Center, whence every way the Quality is diffuſed, in the right Lines A B, A C, A D, and innumerable others ſpread indefinitely through the whole Space. I ſay, that the Intenſion of the Quality decreaſes in a Ratio duplicate of that, whereby the Diſtances increaſe; or, which is the ſame thing, its Intenſion at a Diſtance equal to A B is to its Intenſion at a Diſtance equal to the right Line A E, reciprocally in a duplicate Ratio of the Diſtance A B, to the Diſtance A E; that is, directly as the Square of A E to the Square of A B.



Since, from the Hypotheſis, the Quality is propagated in a Sphere every way by right Lines, its Intenſion at any Diſtance from the Center, will be proportionable to the Spiſſitude or Denſity of the Rays at the ſame Diſtance. By Rays I here mean the rectilineal Ways by which the Quality is diffuſed: Now the Rays that at the Diſtance A B are diffuſed thro' the ſpherical Superficies BCDH, will be diſperſed at the diſtance AE, thro' the whole ſpherical Superficies EFGK: but the Spiſſitude of any given Rays are reciprocally as the Spaces they take up; namely, if the Superficies EFGK is double the Superficies BCDH, the Rays at the Superficies BCDH, will have double the Denſity, that the ſame Rays will have at the Superficies EFGK: and if the Superficies EFGK is triple the Superficies BCDH, the Rays likewiſe at the Superficies BCDH will have triple the Denſity that the ſame Rays will have at the Superficies EFGK; and univerſally whatever proportion the Superficies EFGK has to the Superficies BCDH, the ſame proportion will have the Denſity of the Rays at the Superficies BCDH to the Denſity of the ſame Rays, at the Superficies EFGK. And from *Archimed. of the Sphere and Cylinder*, it appears that all ſpherical Superficies are in a duplicate Ratio of their Diameters or Semidiameters: the Spiſſitude therefore, or Denſity of the Rays by which the Quality is propagated to a Diſtance equal to the Diſtance AB, will be to the Denſity of the ſame Rays at a Diſtance equal to AE, reciprocally in a duplicate Ratio of the Semidiameter or Diſtance AB, to the Semidiameter or Diſtance AE. But as it has been ſaid, the Intenſion of the Quality at any given Diſtance, will always be as the Spiſſitude of the Rays by which it is propagated to that Diſtance; therefore the Intenſion of a Quality at a Diſtance equal to AB, will be to the Intenſion of the ſame Quality at a Diſtance equal to AE, reciprocally in a duplicate Ratio of the Diſtance AB to the Diſtance AE; that is, directly as the Square of AE to the Square of AB.

WE have demonſtrated this Theorem univerſally, whatever is the Nature of the Quality, ſo that it acts in right Lines; and it hence follows, that the

Intenſions of Light, Heat, Cold, Perfumes, and the like Qualities, will be reciprocally as the Squares of their Diſtances from the Point whence they proceed. Hence alſo may be compared amongſt themſelves, the Actions of the Sun on different Planets: but this is not the Buſineſs of our preſent Deſign.

AFTER the Ratios of the Forces are diſcovered in given Conditions or Suppoſitions, then thoſe Ratios are to be compared with the Phenomena of Nature, that there may be found out, what Conditions of the Forces belong to every ſort of Bodies. But in order to obtain this, many of the Experiments that the third Sect of Philoſophers have delivered down to us, muſt be made uſe of: tho' this ought not to be done without great Caution; for we are well appriſed how fond theſe Gentlemen are of their Theories, how willing they are that they ſhould be true, and how eaſily they deceive both others and themſelves, in trying their Experiments. Such therefore as are produced by all, and which ſucceed upon every Trial, we receive as undoubted Principles or Axioms: as likewiſe we ought ſooner to give credit to thoſe Experiments that are more ſimple and eaſy to be ſhewn, than to thoſe that are more compounded, and difficult to be performed.

LASTLY, We ought to inquire with the antient Atomiſts, and the Followers of the new Philoſophy, what are the Phenomena that may be explained by Matter and Motion, and the known and eſtabliſhed Laws of Mechanicks.

BUT that we may proceed in this Affair with the greater ſafety, and, as much as poſſible, avoid all Errors: we ſhall endeavour to obſerve the following Rules.

FIRST, We ſhall, after the Method of the Geometers, premiſe ſuch Definitions, as are neceſſary to arrive at the Knowledge of Things. It is not to be expected that theſe ſhould be logical Definitions, which conſiſt of the *Genus* and *Difference*; or ſuch as diſcover the intimate Eſſence and ultimate Cauſe of the thing defined: theſe Matters I leave to be diſputed by others; for ingenuouſly to confeſs my own Ignorance, the intimate Natures and Cauſes of Things are not known to me. And whatever acquaintance I have with Bodies or their Actions, I obtained it either by the help of my Senſes, or elſe deduced it from ſome of their Properties, which Properties were diſcovered by the ſame means. I ſhall therefore, inſtead of ſuch a Definition as the Logicians are wont to give, exhibite a Deſcription; whereby the thing deſcribed may be clearly and diſtinctly conceived, and likewiſe be diſtinguiſhed from every thing elſe. Wherefore we ſhall define all things by their Properties, chuſing out one or more of the ſimpleſt, which by Experience we are certain do really belong to the things themſelves; and then from theſe, we ſhall after a Geometrical manner deduce other Properties of the ſame things. Many modern Philoſophers have offended againſt this Rule, who define things not by any Properties which do certainly belong to them, but by Eſſences and Natures which they ſuppoſe to be in the things themſelves. They ſuppoſe it indeed, but at the ſame time it does not at all appear, whether ſuch Natures as they define, are really in the things themſelves. The *Carteſians*, for example, ſay that a Fluid has its Parts in continual Motion, but it does not appear from our Senſes, or Experience, or Reaſon, that ſuch is the Nature of a Fluid: nay, that very Argument which they produce to confirm this their Hypotheſis, we ſhall overthrow by a Geometrical Demonſtration. For they will have it, that the Reſiſtance of a Body moving in a Fluid, is leſs, if the Parts of that Fluid are agitated by an inteſtine Motion, than if there was no ſuch Motion in the Fluid; the contrary to which, we ſhall demonſtrate, when we come to treat of the Reſiſtance of Fluids.

BUT the Writers of the Mathematical Philoſophy have followed a much better courſe, who take their Definition of a Fluid from its moſt obvious Property. A Fluid, they ſay, is a Body whoſe Parts yield to the ſmalleſt Force or Impreſſion, and by receding, are eaſily moved in reſpect to each other. From which Definition they deduce many beautiful Theorems that are of Service in the Occaſions of Life; whereas the *Carteſian* Philoſophers from theirs have produced nothing either certain or ſolid, much leſs uſeful.

SECONDLY, While we are inveſtigating natural Truths, it will be of advantage to conſider only the Conditions that were ſuppoſed at firſt, abſtracting for a time from all other Conſiderations whatever. For as the human Mind is finite, ſo if it be diſturbed by too great a number of things at once, it will by that means be rendered incapable of making any Diſcoveries. The Mechanical Writers have obſerved this Rule in their comparing the Spaces paſſed over by two Bodies in motion: for they look upon the Bodies in this caſe as Points, abſtracting from the Conſideration of their Magnitude, Figure, and Colour, which make no alteration at all in reſpect to the Length they have moved.

THIRDLY, It is neceſſary to begin with the moſt ſimple Caſes firſt; and having once ſettled them, we may thence advance to ſuch as are more compounded. So the ſame Mechanical Philoſophers at firſt ſuppoſe the Motion of Bodies to be *in vacuo*, or in a Medium that has no Reſiſtance; and having determined the Laws of Motion in that Caſe, they thence proceed to inveſtigate the Laws of Reſiſtance, and laſtly to diſcover what Changes are thereby likely to ariſe to

Bodies in motion. For the leſs the Bodies are reſiſted by the Medium they move in, the leſs will the Laws of the Motion of Bodies in that Medium differ from the Laws before found out. So likewiſe in Hydroſtaticks, we conſider the Fluid as having no Tenacity or Coherence of its Parts, but that they are ſeparable by the leaſt Force whatſoever: and it is on this ſuppoſition, that we determine the Poſitions of Bodies immerſed in ſuch a Fluid, and the Preſſures of the Fluid on ſuch Bodies. Tho' perhaps there is not in Nature ſuch a Fluid, as has no Coheſien of its Parts; wherefore the Variation or Diſagreement in the Laws of a Fluid whoſe Parts do cohere, from thoſe of a Fluid which we ſuppoſe to have no Coheſion in its Parts, ought to be inveſtigated; and if that Tenacity or Coherence of Parts in the Fluid be but ſmall, the Diſagreement from the Laws firſt diſcovered, will be but ſmall, and ſcarce diſcernible.

MOST of the Theoriſts offend againſt this Rule; who neglecting, or not thoroughly underſtanding the firſt and more ſimple Principles of the mechanical Philoſophy, at the very firſt ſtroke attempt the moſt difficult Problems, and raſhly enough endeavour to ſhew how a World, a Planet, or an Animal might be formed. Theſe are not unlike ſome young Beginners in Geometry, who before they are well acquainted with the Elements of that Science, preſently ſet upon the Quadrature of the Circle, the Triſection of an Angle by the means of right Lines and Circles, the Duplication of the Cube, and the like: ſo our Theoriſts, not having laid a good Foundation, raiſe up but a weak Superſtructure; whence it is not to be wondered, if their great Building immediately ſinks, not without the diſgrace of the Architects. But thoſe who philoſophize aright, ought to take another courſe, and proceed in a quite different Method; and tho' they do not pretend to form a World, an Earth or a

Planet, yet they may be able to lay down a ſure Foundation, and firm Principles of the Mechanical Philoſophy, and explain the Phenomena, which are thence to be deduced.

LECTURE II.

Of the Solidity and Extenſion of Bodies.

WE ſhall not here give a Definition of Body, taken from its intimate Nature or Eſſence, wherewith we are not perfectly acquainted, and perhaps never ſhall be: but according to a Rule laid down in the former Lecture, we ſhall define it by ſome of its Properties, that diſtinguiſh it from every other Being whatever. We therefore ſay, That a Body is, *what is extended, ſolid, and capable of Motion.*

THERE is no one, I ſuppoſe, ſo ſlow of Apprehenſion, as not readily to perceive, that every finite Body has Bounds or Terminations, which Terminations we call Superficies, and of theſe ſome one is diſtant from its Oppoſite; and again this Superficies (ſince it is not infinite) has Extremes, which we call Lines, and betwixt which there muſt of neceſſity be ſome diſtance. And moreover there are Terminations of theſe Lines, which we name Points, betwixt which alſo there is an Interval. From all theſe Diſtances taken together, we acquire a diſtinct Idea of Extenſion into its triple Dimenſion. For the Diſtance betwixt two oppoſite Surfaces of the ſame Body, is called its Thickneſs or Depth; the Diſtance betwixt two oppoſite Lines of the ſame Superficies, is ſaid to be its Breadth; and the Diſtance betwixt both the Extremes of a Line, is named its Length. There is no Body that does not partake of this triple Dimenſion, and however ſmall we ſuppoſe it to be, it is neceſſary that it ſhould have Length, Breadth, and Thickneſs: and that which is in a Body, and is deſtitute of all theſe, is not a Body, but a Point; neither is it a Magnitude, but the Beginning or Termination of a Magnitude.

SOLIDITY is that Property of a Body, whereby it reſiſts all other Bodies that preſs it on every ſide; and whereby, whilſt it poſſeſſes any place, it hinders all other Bodies from entering into that place, tho' they are forced againſt it ever ſo violently. So, for example, if a Body is held betwixt one's hands, tho' it is preſſed with ever ſo great a Force, it prevents one's hands from coming to a mutual Contact.

THIS is that Property, which moſt of the *Peripateticks* are wont to call Impenetrability; that is, whereby two Bodies are not ſuffered to be in the ſame place at once, or to penetrate each other: tho' I rather chuſe, with an illuſtrious Philoſopher of our Age, to call it Solidity. This Property ſeems to be ſo eſſential to all Bodies, that there is no other Being in Nature to which it can belong: for tho' there are other ſorts of Magnitudes, corporeal Magnitude alone admits of Solidity; all other Quantities, or indeed ſuch as are no Quantities, as Points, can penetrate, unite, and be in the ſame place. For if two Spheres meet each other, in the place of Concurſe there will be a Point of one united with a Point of the other, or they will agree, or will be in the very ſame Point of Space: So if two Cubes are equal, the one may be ſo placed upon the other, that two of their ſquare Surfaces may agree; namely, the Sides of one of the Squares may coincide with the Sides of the other Square; and the Angles of one be united with the Angles of the other, whence theſe Quantities mutually penetrate each other, and are in the ſame place, which is impoſſible to happen in Bodies.

YOU may eaſily perceive, that we uſe the word *Solidity* in a ſenſe very different from that of the Geometers, who ſuppoſe that Solids may mutually penetrate each other. As, for Example, when *Euclid*, in the eleventh Book of his Elements, demonſtrates the Equality of two Parallelepipedons, placed upon the ſame Baſe, and betwixt the ſame parallel Planes; it follows, that two different Parallelepipedons ſo placed muſt neceſſarily penetrate each other: whence it is manifeſt, the Geometers ſuppoſe their Solids penetrable. The Geometers therefore take the word *Solidity* in a ſenſe different from that of the Philoſophers, neither do they oppoſe their Solids to penetrable Magnitudes, but to Planes, or Surfaces, to plane Angles, and Lines; for every thing is reckoned ſolid by them, that has three Dimenſions.

BUT the Solidity of Bodies is of another ſort, which as we have ſaid that it belongs to Bodies alone, ſo it is common to every Species of Bodies, whether they are fluid, or hard and fix'd, or eaſily moved and yielding to any Force, or very heavy, or whether they have but little Weight, or are abſolutely light, if ſuch Bodies can be in Nature; for a Drop of Water or a Particle of Air remaining fixed betwixt any two Bodies, does not leſs hinder the Contact of thoſe two Bodies, than the hardeſt Metal or a Diamond would do.

LASTLY, By this Property, Body is diſtinguiſhed from another kind of Extenſion, which we conceive to be penetrable, which we call Space, and wherein we behold all Bodies to be placed and moved, at the ſame time regarding that as immoveable.

THE *Carteſians*, who define Body by its Nature, which they make to conſiſt in Extenſion alone, allow of no Space or Extenſion that is not corporeal; but ſince we have, or at leaſt imagine we have, an Idea of Space diſtinct from the Idea of Body; they certainly offend againſt the Laws of good Method, who place the Nature or intimate Eſſence of Body in any of its Attributes, which, we are not ſure, do only belong to it.

But the *Carteſians* ſay, that the Nature of Body cannot conſiſt in any other of its Attributes, ſince neither Hardneſs, or Colour, or Weight, or Figure, or Taſte, or any of its ſenſible Qualities, can conſtitute its Eſſence. For all theſe Attributes may be taken from the Body, and yet the Nature of Body will remain; but take away Extenſion, and the corporeal Being is immediately deſtroyed: therefore it is neceſſary that the Nature of Body ſhould be placed in Extenſion alone.

This is the Argument of *Cartes* himſelf, tho' it is unworthy a Philoſopher; for nothing can thence be deduced, unleſs this, that thoſe ſenſible Qualities that he mentions, are not of the Eſſence of Body, and that Extenſion is an Attribute neceſſary and eſſential to it. But what then? Cannot the ſame univerſal Attribute agree to two different Species of things? Is it neceſſary that all things that have the ſame Attribute, muſt have the ſame Nature, and Eſſence? If this be ſo, there will be no Diſtinction, no Diverſity in things. Tho' therefore Space and Body have one and the ſame eſſential Attribute common to them both, yet they are very different things; and there are other eſſential Attributes, peculiar to each, whereby they are ſufficiently diſtinguiſhed.

In the firſt place, the above-deſcribed Solidity is proper to Bodies only, and ſo eſſential to all of them, that you cannot ſo much as ſeparate it from them in your Imagination, but at the very ſame time you deſtroy that very Idea which you had formed of

Body: wherefore if the Eſſence and intimate Nature of Body is to be placed in ſome one Attribute, Solidity certainly has a much better pretence to be that Attribute than Extenſion; eſpecially when there ſeems to be a Being different from Body, which we call Space, to which likewiſe Extenſion appertains: at leaſt, the contrary does not yet appear.

BESIDES that we have the Idea of this Space altogether diſtinct from the Idea of Body, each of them ſeems to be endued with Attributes, not only diſtinct and proper to themſelves, but ſo contrary, that it is impoſſible to imagine they could be inherent in the ſame Subject. For we conceive Body as ſolid or impenetrable, diviſible and capable of Motion, whoſe Parts may be eaſily diſunited, ſeparated and removed from one another to any diſtance whatever: one Body may oppoſe another Body that is moving, it may ſtop, or at leaſt diminiſh its Motion; a Body likewiſe may communicate its Motion to another Body that is at reſt, or moves with a leſs force towards the ſame or oppoſite Parts, and carry it along with it.

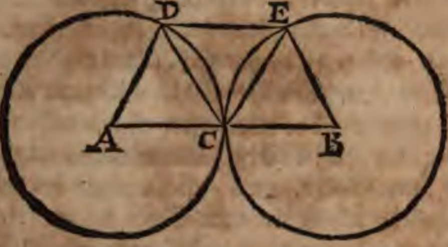
ON the contrary, we conceive Space to be that, wherein all Bodies are placed, or, to ſpeak with the Schools, have their *Ubi*; that it is altogether penetrable, receiving all Bodies into itſelf, and refuſing an Ingreſs to nothing whatſoever; that it is immoveably fixed, capable of no Action, Form or Quality; whoſe Parts it is impoſſible to ſeparate from each other, by any Force however great; but the Space itſelf remaining immoveable, receives the Succeſſions of things in motion, determines the Velocities of their Motions, and meaſures the Diſtances of the things themſelves. Theſe ſo diſagreeing and repugnant Attributes of Space and Body, it is impoſſible ſhould belong to the ſame Subject.

THE *Carteſians* perhaps may reply, That this Idea, which we have here given of Space diſtinct from Body, is altogether imaginary and chimerical, the like to which cannot, they ſay, be made to exiſt in the Nature of things, by any Power whatſoever. But we are ready to demonſtrate againſt the *Carteſians*, that in truth there is given a Space diſtinct from Body; or, in other Words, that Space and Body are not the ſame things: but we ſhall firſt of all obſerve, that we do not now intend to prove the real Exiſtence of Space void of all Body, for that we ſhall do in another Lecture; we ſhall at preſent be content, only to ſhew the Poſſibility of it.

LET us ſuppoſe then any Veſſel, and let it at firſt be filled with Air; then let the Air contained in this Veſſel be exhauſted, or, if you will, annihilated by a Divine Power, and let all other Bodies whatever be hindered from entering into its Place: I would now aſk, whether in this Caſe there is not given a Space void of all Bodies? All that Body which was within the Veſſel is deſtroyed, the Ingreſs of any other Body is prevented, and the Form of the Veſſel is ſuppoſed to be preſerved; it ſeems therefore to be neceſſary that a Vacuum, or a Space not replete with Body, is given. The *Carteſians* may anſwer, That on theſe Suppoſitions the Sides of the Veſſel muſt fall in, and neceſſarily approach one another. But ſince, according to the *Carteſians* themſelves, no Body can move of itſelf; and ſince, by Hypotheſis, there is no other Body that forces the Sides of the Veſſel towards one another; it therefore follows, that the Sides will not approach one another. They will ſay, perhaps, that the Air diffuſed every way, and preſſing on all hands the Sides of the Veſſel, will cauſe that Motion. But ſince the Preſſure of the Air is of a finite Force, the Strength of the Veſſel may be ſufficient to balance that Preſſure, and conſequently its form will be preſerved. But let us grant that its Sides may fall in, I demand, what Body will ſucceed into their place? (They will anſwer,) The Air. What Body will poſſeſs the Space left by that Air? (Perhaps they will ſay,) More Air? But we muſt ſtop ſomewhere, and at length come to ſome Body, into whoſe place no other Body will ſucceed; for it is abſurd to go on thus *ad infinitum:* therefore in that caſe, it is neceſſary to ſuppoſe a *Vacuum.*

But we ſhall alſo ſhew, by an invincible Demonſtration taken from Geometry, at leaſt the poſſible Exiſtence of a Space void of all Body: towards which Proof, we ſhall premiſe the two following Aſſertions, or Axioms, whoſe Evidence no Philoſopher will call in queſtion. The firſt is, That no Body or Portion of Matter ſtands in need of the Exiſtence of another Body, in order to its own Exiſtence. As, for example, a Sphere may exiſt, whether any other Body exiſts or not: this evidently follows from the Nature of Subſtance. Secondly, Any Body, eſpecially if it be hard, can preſerve its Figure, if there are no external Bodies or no Agents, which endeavour to induce an Alteration in it. Certainly it muſt be confeſſed, that God can preſerve any Body in the ſame State and Poſition; and whatever may happen externally, he can notwithſtanding continue the Figure of that Body immutable.

Since therefore one or two Spheres may exiſt, without the Exiſtence of any other Bodies; let us ſuppoſe all other Bodies, except two Spheres, to be annihilated by God; or rather let us ſuppoſe all the Matter in the Univerſe to be amaſſed into two Spheres, which may be repreſented by two Circles, whoſe Centers let be A and B: and ſince no other Body is ſuppoſed to exiſt, thoſe ſpherical Bodies may preſerve their Figure, there being no other external



Body either to deſtroy or alter it. Theſe two Spheres then are either contiguous or diſunited; if they are disjoined, there will be an intermediate Space replete with no Body, and therefore all Space is not Body: But if theſe Spheres touch one another, it is neceſſary that they touch one another in one Point only, by the Elements of Geometry: therefore betwixt the other Points of the Spheres there is ſome Diſtance, that is, ſome Space will lie between. For out of the Point of Contact let there be taken any two Points, as D and E; if betwixt theſe Points there is no Space, that is, no Diſtance, theſe Spheres will touch one another in theſe Points, which is impoſſible.

AGAIN, That there is a Space void of all Body, may be thus ſhewn. Let us ſuppoſe two Spheres, in which all the Matter of the Univerſe is accumulated, and that theſe two Spheres are equal; in both which let be accommodated the right Lines CD, CE, each equal to the Semidiameter of either Sphere, join DE; this right Line will be equal to the Semidiameter of one of the Spheres. For, draw AD, BE, and becauſe in the equilateral Triangles ACD, BCE, the Angles ACD, BCE, are each the third part of two right Angles, the Angle DCE will be equal to the third part of two right Angles; for all the Angles at the Point C, make two right ones: whence ſince DC, CE, are equal, the Angles CDE and CED will be likewiſe equal; but taken together, they make two Thirds of two right Angles: wherefore each is one third part of two right Angles, whence the Triangle DCE is equi-angular; and conſequently, DE will be equal to the Semidiameter of each Sphere, nor in this caſe can it be more or leſs. In like manner, betwixt any other Points of the Spheres out of that of Contact C, there will be ſome Diſtance, having a determinate Ratio to the Diameter of the Spheres; and therefore there will be betwixt theſe Spheres a certain and determinate Space, not replete with Matter: but into that Space may be admitted a Body, whoſe Dimenſions agree with theſe Diſtances; but what has greater Dimenſions, cannot by any Power whatever be placed within that Space. Whence, ſince theſe Properties demonſtratively agree with the Space here ſpoken of, and that Space may really exiſt, tho' no one ſhould think at all, it evidently follows againſt the *Carteſians*, that the Idea we have of Space, is not chimerical or imaginary; for what is chimerical, can have no exiſtence out of the Intellect.

WE conclude therefore, that there is in reality a Space diſtinct from all Body; which is as a univerſal Receptacle, wherein all Bodies are contained and moved. But what is the Nature of this Space, whether it is any thing poſitive, actually extended in itſelf, and endued with real Dimenſions, or whether its Extenſion ariſes from the Relation of Bodies exiſting in it, ſo that it may be a mere Capacity, Ponibility, or Interponibility, as ſome love to expreſs themſelves, and to be reckoned in the ſame Claſs of Being with Mobility and Contiguity; or whether this our Space is the divine Immenſity itſelf, which is through all and in all, or whether it is created or uncreated, finite or infinite, dependent or independent on God; theſe things we do not here enquire into, but leave them to be diſputed by the Metaphyſicians. It ſerves our turn to be able to explain ſome of its Properties, and to eſtabliſh and demonſtrate its Diſtinction

ſtinction and Nature to be different from that of Body: They who would have more, may conſult the Philoſophers.

LECTURE III.

Of the Diviſibility of Magnitude.

ALTHOUGH it may be demonſtrated by many Arguments, that Space is really diſtinct from Body, and we have already produced ſome, that ſeem to be unanſwerable; yet both theſe things agree in this, that Extenſion is a univerſal Attribute, neceſſarily and eſſentially belonging to both of them. Before therefore we proceed any farther, it may not be foreign to the purpoſe to explain ſome general Affection of Extenſion; as for example, its Diviſibility.

THIS Property of Extenſion appertains and neceſſarily adheres to all Species of Magnitude, as well to Lines as Surfaces, and as well to Space as to Body. By Diviſibility we would not here be underſtood to mean an actual Separation of Parts from one another, which ſuppoſes Motion, which indeed the Nature of Space does not admit, nor do the Demonſtrations borrowed from Geometry prove ſuch a Separation; but the Diviſibility that we here endeavour to evince, is only the Reſolution of any Magnitude into its Parts, or their Diſtinction and Aſſignment. As, for example, when *Euclid*, in the ninth Propoſition of his third Book, teaches how to cut a rectilinear Angle into two equal Parts, he does not in that Method undertake to ſhew, how one of the equal Parts being ſeparated from the other, recedes and is placed from it at a given Diſtance; but only delivers a Method, whereby a Line may be drawn, dividing the Angle in ſuch a manner into two other Angles, that the Angle which lies on one ſide of this Line, ſhall be equal to the Angle that lies on the other ſide the ſame Line. So likewiſe when, in the following Propoſition, he teaches how to biſect any right Line, he only ſhews how to aſſign a middle Point, dividing the given right Line into two equal Parts, which Point is the common Termination of both the Parts; namely, where one of the equal Parts ends, and the other begins. This Reſolution of Magnitude into its Parts, is ſo intimate and eſſential to it, as that which has no Parts, as, for inſtance, a Point, is not ſaid to be a Magnitude, but the Beginning or End of Magnitude: nor can any Magnitude be produced by any Number of Points, tho' infinite; for every Magnitude is not compounded of Points, but Parts, that is, other Magnitudes of the ſame kind, whereof every one is conſtituted of other Parts, and each of theſe is ſtill made up of others, and ſo on *in infinitum:* nor can we ever arrive at a Magnitude ſo ſmall, but it may be yet farther divided into Parts; nor is there given, in any Species of Magnitude, an abſolute *Minimum*, but whatever is divided, is ſtill farther diviſible into Parts. This conſtant farther Reſolution of Matter into Parts, is by the Philoſophers called its *Diviſibility in infinitum*; and that very truly, ſince there cannot be aſſigned any Quantity of Matter ſo minute, and any finite Number ſo great, but that the Number of Parts compoſing that Magnitude, that is, into which it may be reſolved, ſhall be greater than that Number, how large ſoever it be: *for we call that Infinite, which exceeds any Finite.*

BUT becauſe this infinite Diviſibility of Matter can be demonſtrated by Arguments taken from Geometry, and ſince there are now-a-days ſome Philoſophers who attempt to baniſh Geometry out of Phyſicks, by reaſon they are ignorant of that divine Science; and as theſe Gentlemen would be reckoned amongſt the moſt Learned, they leave no means untried, whereby, tho' in vain, they may overturn the Force of theſe Demonſtrations: It will be therefore neceſſary, before we produce our Geometrical Arguments, to eſtabliſh their Strength, and to anſwer ſome Objections.

As amongſt the Philoſophers of this Claſs, the famous *John Baptiſt du Hamel*, the Author of the *Burgundian* Philoſophy, is of the greateſt Eminence; we ſhall produce his Opinion on this Subject. He ſays then, that Geometrical Hypotheſes are neither true nor poſſible, ſince neither Points, nor Lines, nor Surfaces, as the Geometers conceive them, do truly exiſt in the Nature of Things; and therefore that the Demonſtrations that are produced from theſe, cannot be applied to things actually exiſting, when none of theſe exiſt any where but in our Ideas. He deſires therefore the Geometers to keep their Demonſtrations to themſelves, and not to make uſe of them in Philoſophy, becauſe, according to him, they ſpread over this Science not Light, but Darkneſs. I admire at the Unſkilfulneſs of this otherwiſe moſt Learned Perſon, in this Affair: he might certainly with the ſame Juſtice take away all Phyſical Suppoſitions whatſoever, ſince Geometrical Hypotheſes are equally certain and equally poſſible and real, as are thoſe, which he calls Phyſical. Certainly if Body exiſts, there muſt of neceſſity exiſt real Points, real Lines, and real Surfaces, even ſuch as are conceived by the Geometers; as we can eaſily make appear. For if Body be given, that, ſince it is not infinite, has its Terminations; but the Terminations of Body are Surfaces, and thoſe Terminations have no Depth: for if they had, they would thereby be Bodies, which Bodies would have ſtill other Terminations, which would be Surfaces, and therefore there would be a Superficies of a Superficies. Either then this Superficies is deſtitute of all Depth, or not: if the firſt, we have what we require; if the latter, we come again to another Superficies, and ſo we ſhould proceed *in infinitum*, which is abſurd. Wherefore we muſt conclude, that thoſe Terminations are deprived of all Depth, and are therefore true Surfaces, and as they are conceived by the Geometers without any Depth, or ſuch as have only Length and Breadth to conſtitute their Eſſence.

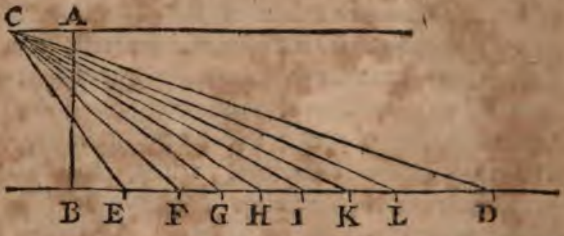
AGAIN, Since this Surface is not infinite, it is likewiſe bounded by its Terminations; but thoſe Terminations are called Lines, which have really no Breadth: for otherwiſe they would be Surfaces, and would have alſo their Terminations, which we ought to conceive at leaſt deſtitute of all Breadth; for, as we ſaid before, there cannot be given a Progreſſion *in infinitum*; whence there are really given Lines, which are only extended in Length, without any Breadth. After the ſame manner, Lines alſo have their Terminations, which are called Points, to which belong neither Length, nor Breadth, nor Depth. Wherefore if Body may be ſuppoſed to exiſt, it neceſſarily follows, that Geometrical Surfaces, Lines, and Points, may be ſaid not only as poſſible to be, but alſo to be actually exiſting.

BUT it will be anſwered, That theſe Points, Lines, and Surfaces are not material. What then? Who ever aſſerted that a Mathematical Point was Matter? Who ever fancied a material Superficies? If it was material, it would have its Superficies or Termination: but the Superficies of a Superficies, who ever imagined? However, tho' neither Surfaces, nor Lines, nor Points are real Matter; yet they exiſt or may exiſt in it, as its Modes, Terminations, or Accidents: juſt after the ſame manner, as Figure is not Body itſelf, but only its Affection, whereby it is contained under given Terminations, and this has real Properties wholly diſtinct from thoſe of Body.

AND again our ungeometrical Philoſophers may object, That there is not in Nature a perfectly plane Superficies, a Body perfectly ſpherical, ſuch as the Geometers feign, nor any Curve perfectly circular. But how come they to know all this? Have they ſeen all the Bodies that are in the Univerſe, and view'd them thro' a Microſcope? Perhaps they will ſay, that a Superficies cannot be plane or ſpherical, becauſe in theſe Figures there is a Contradiction and an Impoſſibility. But we deſire they would be pleaſed to make out this Contradiction: All Bodies of neceſſity are terminated by ſome Figure, plane or ſpherical Surfaces are the moſt ſimple and eaſieſt to be conceived of any; where then lies the Repugnancy, that it ſhould be impoſſible for a Body to be contained under theſe Surfaces? I believe there is no one who is acquainted with the Elements of Geometry, but perceives more the Nature and Properties of theſe Figures, and knows more of their Affections, than all the Philoſophers of this ſort put together: but none of thoſe ever found any Repugnancy in theſe Figures; no Geometer ever ſuſpected theſe Contradictions in the Nature of Figures: on the contrary, ſo many beautiful Properties of thoſe Figures diſcovered and demonſtrated by the Geometers, evince their Poſſibility, for of an impoſſible thing there can be no real Property, no Demonſtration. It remains therefore, that they acknowledge theſe Figures as poſſible; and if they are poſſible, it is in the power of God to form out of Matter, Bodies having ſuch Surfaces. Let us therefore ſuppoſe two Bodies, whereof one is terminated by Planes, the other by a ſpherical Superficies; if therefore the ſpherical Superficies is placed on the Plane, it will touch it; but it will touch it only in one, and that an indiviſible Point, or in a Point that has no Parts, by *Cor. Prop.* 2. *El.* 3. and therefore in that caſe there will be given a true Point. But farther, let us ſuppoſe this ſpherical Body to be moved on the plane Superficies, or to be carried along without any Rotation about its Axis, inſomuch that the Point touching the plane Superficies may be always found in the ſame Plane; then the Tract which that Point deſcribes by its motion, will be a Mathematical Line, without any Breadth; and if the Diſtance betwixt any two Points in that Plane is the ſhorteſt, there will be generated by that Motion a right Line; but if otherwiſe, either a Curve, or one compounded of right Lines, or partly of theſe, and partly of thoſe, will be deſcribed. Points therefore, Lines, and Surfaces, even ſuch as the Geometers conceive or feign, are poſſible; which was to be ſhewn. And their Poſſibility may be demonſtrated after innumerable other ways; but we are weary of dwelling on theſe Trifles. We ſhall only farther obſerve, that the Diſtance betwixt any two Points of two Bodies, will be their given and determinate Diſtance: As, for example, the determinate Diſtance betwixt the Centers of the Sun and a fixed Star, is that which is meaſured by a right Line lying betwixt thoſe two Points; which, of all the Lines that can be drawn from one Point to the other, will be the ſhorteſt, and paſſed over in the leaſt time with a given Velocity: this Diſtance, I ſay, will remain the ſame, whatever ſhall happen to be the Figure of the intermediate Body, whether it is bounded by Planes, or contained under ſpherical Surfaces, or laſtly if there ſhould be no intermediate Body, and nothing but Space lie betwixt;

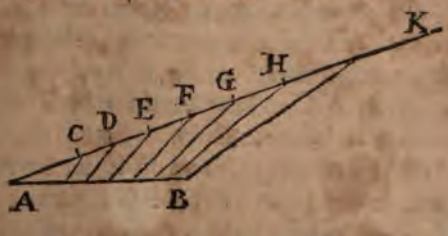
the Line will remain the ſame in Magnitude and Poſition, as long as the Centers of the Bodies remain unmoved.

HAVING now ſettled theſe Principles, we return to our Purpoſe; which was to demonſtrate that all Extenſion, whether corporeal or incorporeal, was diviſible *in infinitum*, or had an infinite Number of Parts; which we ſhall endeavour to prove by many invincible Arguments. Of which, this ſhall be the firſt: Let AB repreſent a right Line, I ſay it is diviſible into Parts exceeding any finite Number whatever.



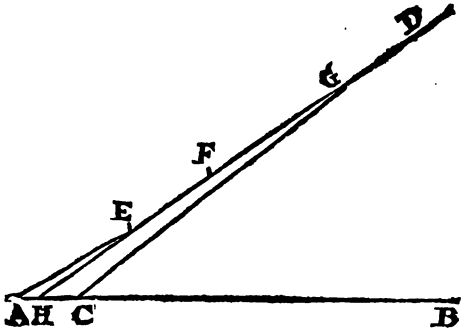
THROUGH A let be drawn any right Line AC, and parallel to it let be drawn through B the right Line BD, and in AC let there be taken any Point, as C: if therefore the right Line AB is not diviſible into an infinite Number of Parts, let it be diviſible only into a finite Number of Parts; and let that Number, for example, be ſix. In the Line BD on the ſide oppoſite to C, let there be taken any Number of Points exceeding ſix; for Example, the Points E, F, G, H, I, K, L, and let there be drawn by the firſt Poſtulate of *Euclid*, CE, CF, CG, CH, CI, CK, CL. Theſe thus drawn, divide the right Line AB into as many Parts as there are right Lines; for if they do not, then ſome of the right Lines interſect AB in one and the ſame Point: but all of them interſect one another in the common Point C, whence ſome two right Lines will cut one another twice, or will have the ſame common Segment; both which is contrary to an Axiom in the *Elements*. A B is therefore divided into as many different Parts, as there are right Lines; but there are as many right Lines, as there were Points taken in the right Line B D: wherefore ſince there were taken more Points than ſix, the right Line A B is diviſible into more Parts than ſix. After the ſame manner, how great ſoever the Number aſſumed ſhall be, it may be ſhewn that the Line A B is diviſible into a Number of Parts greater than that Number; namely, by taking in the right Line B D a greater Number of Points, (which may be eaſily done, ſince no finite Number is ſo great, but a greater may be aſſumed, and that in any given Ratio of a greater Inequality) and by drawing right Lines from the Point C to the Points taken in the right Line B D: for theſe right Lines will divide the right Line A B into as many Parts, as there are right Lines, and therefore into more Parts than the Number firſt aſſumed (how great ſoever it was) contains Units; and conſequently the right Line A B is diviſible into more Parts than can be expreſſed by any finite Number, and therefore it is diviſible *in infinitum*. *Q. E. D.*

THE ſecond Argument. Let A B repreſent any right Line, I ſay it is diviſible into an infinite Number of Parts; for if it is not diviſible into an infinite Number of Parts, let it be diviſible into a



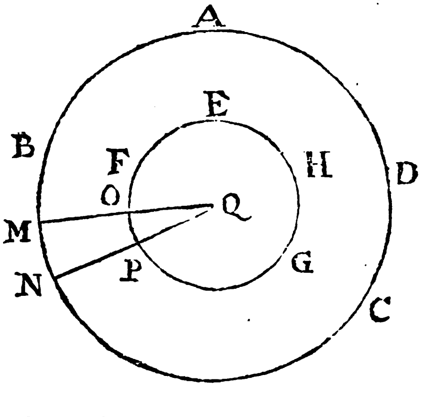
finite Number of Parts; and let that Number be, for example, five. Let any right Line A K be drawn, making any Angle with A B, and in it, produced if neceſſary, let there be taken as many Points as you pleaſe above five, which let be C, D, E, F, G, H, K; join K B, and through the Points C, D, E, F, G, H, let right Lines be drawn parallel to K B: theſe will neceſſarily divide the right Line A B into as many Parts as there are right Lines; for if they do not, more right Lines muſt concur in one and the ſame Point: but they cannot concur, ſince they are parallel; wherefore each right Line will interſect the right Line A B in a different Point, and all will divide the right Line A B into as many Parts, as there were right Lines drawn parallel. But there were more drawn than five, therefore the right Line A B will be divided into more Parts than five. And the ſame may be affirmed of any other Number. Wherefore no Number is ſo great, but the Number of Parts the right Line is diviſible into, is ſtill greater; and conſequently the right Line A B is diviſible *in infinitum*.

THIRDLY, If Quantity is not diviſible *in infinitum*, it muſt be diviſible into Parts, that are not farther diviſible; but there is no Part that cannot be ſtill farther divided, becauſe there can be given no Quantity ſo ſmall, but there may be ſtill taken a ſmaller, and that in any given Ratio of leſſer Inequality. For let A B be a right Line, and let A C be an exceeding ſmall Part of it; I ſay, there may be a Line leſs than A C in any Ratio of leſs Inequality, as, for example, one to three. From the Point A draw any right Line A D, and in it let be taken the equal right Lines A E, E F, F G; join G C, and through E draw E H parallel to G C, the right Line A H will be a third Part of A C: The Demonſtration thereof is manifeſt from the *ninth Propoſition of the ſixth*



ſixth Book of the Elements. And therefore the right Line A C will not be the leaſt that can be taken. The ſame may be demonſtrated of any right Line whatſoever; conſequently there is not in Nature an abſolute *Minimum.*

AGAIN, If Quantity was compos'd of Indiviſibles, many Abſurdities would thence follow: for let there be, for example, two concentrick Circles, ABCD, EFGH, and let the Circumference of the



greater be divided into its indiviſible Parts, and let be drawn from the Center Q to each of theſe Parts, the right Lines QOM, QPN, which will divide both the Circumferences into an equal Number of Parts, but the greater Circumference ABCD was divided into its ſmalleſt Parts; and therefore the leſs Circumference EFG will conſiſt of as many Indiviſibles, or ſmalleſt Parts poſſible, as the greater Circumference ABC: And conſequently ſince an Indiviſible is equal to an Indiviſible, the Circumference EFGH will be equal to the Circumference ABCD, a leſs to a greater, which is abſurd.

LASTLY, From this Compoſition of Quantity of Indiviſibles, there can be no incommenſurate Magnitudes; which is contrary to what the Geometers frequently demonſtrate. For if all Magnitude conſiſted of Indiviſibles, an Indiviſible would be an adequate and common Meaſure of all Magnitudes of the ſame kind; for it would be exactly contained ſome number of times in all, and therefore all Magnitudes would have a common Meaſure, and the Side of a Square would be commenſurate to its Diagonal; contrary to *the laſt Propoſition of the tenth Book of* Euclid*'s Elements.*

THERE might be innumerable other Demonſtrations produced, to ſhew the infinite Diviſibility of Quantity, and entirely to overthrow the Hypotheſis of Indiviſibles. But what occaſion is there for more? ſince the Arguments hitherto alledged, have not leſs force to compel the Aſſent, than any Demonſtration in *Euclid*'s *Elements*; inſomuch that it is as impoſſible to weaken them, as to deſtroy the Fundamentals of Geometry, which no Age or Sect of Philoſophers has been ever able to effect.

THAT therefore the Philoſophers may avoid the Force of theſe Arguments, they diſtinguiſh betwixt a Mathematical and a Phyſical Body. Being compelled by the Force of Demonſtration, they readily allow a Mathematical Body may be diviſible *in infinitum*;

nitum; but they deny that a Phyſical Body can be always reſolved into ſtill farther diviſible Parts. But what, I would know, is a Mathematical Body, but ſomething extended into a triple Dimenſion? Does not Diviſibility belong to a Mathematical Body, by reaſon it is extended? But a Phyſical Body is extended after the ſame manner: wherefore ſince Diviſibility depends on the Nature and Eſſence of Extenſion itſelf, and owes to it its Origin, it is neceſſary that it muſt agree to all Extenſions, whether Phyſical or Mathematical. For, to uſe a Logical Expreſſion, whatever is predicated of any *Genus*, is predicated of all the *Species* contained under that *Genus*.

THERE is yet another Diſtinction amongſt the Philoſophers, not unlike the former; whereby they own that every Body is mathematically diviſible *in infinitum*, but they deny that it is phyſically ſo. If theſe Words have any Meaning, it is certainly this: They acknowledge that a Body is mathematically, that is, really and demonſtratively, diviſible *in infinitum*, but they deny it to be ſo phyſically, or according to their falſe *Hypotheſis*: and ſo they have a Diſtinction, againſt which nothing can be replied.

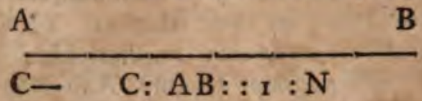
BUT becauſe the Philoſophers, againſt whom we diſpute, are not acquainted enough with Geometrical Demonſtrations, and therefore do not eaſily perceive their Evidence; before we end this Lecture, we ſhall produce one Phyſical Argument taken from Motion, for the infinite Diviſibility of Quantity: namely, If Quantity conſiſted of Indiviſibles, it would follow, that all Motion would be equally ſwift, nor would a ſlow Snail paſs over a leſs Space in the ſame Time than the ſwift-footed *Achilles*. For let us ſuppoſe *Achilles* to run very ſwiftly, and the Snail to creep ſluggiſhly along; if Extenſion conſiſted of Indiviſibles, the Snail could not in any given

Time paſs over leſs Space than *Achilles:* for if in a Moment's Time *Achilles* paſſes over an indiviſible Space, the Snail cannot in the ſame Moment of Time paſs over leſs Space; by reaſon, from the Hypotheſis, there cannot be a leſs. For one Indiviſible cannot be leſs than another, therefore it will paſs over an equal Space. The ſame may be ſaid of any other Moment of Time: Therefore the Spaces paſſed over by them both will be equal; and conſequently the ſwift-footed *Achilles* cannot paſs over more Space than the ſloweſt Snail: which is abſurd. Other Abſurdities of the like ſort, may be deduced from the ſame Hypotheſis of Indiviſibles; but what we have already ſaid, is ſufficient.

LECTURE IV.

Wherein the Objections uſually brought againſt the Diviſibility of Matter, are anſwered.

WE have hitherto produced ſuch Arguments, as by their help we have ſufficiently demonſtrated the continual Diviſion of Matter into an infinite Number of Parts: it now remains, that we anſwer to the Objections or Cavils of the Philoſophers. For there are not a few of them, who labouring under I know not what Obſcurity of Ideas, and not clearly enough perceiving the Evidence of the Arguments we have here made uſe of, produce Arguments of their own againſt a thing ſo manifeſtly true, and to which they are alſo pleaſed to give the Title of Demonſtrations. But as I have peruſed many of their Books, I never lit on any thing in all their Writings upon this head, that had ſo much as the very Appearance of Reaſon: they were ſo far from coming up to Demonſtrations, that no Geometer, however clear-ſighted he was, could ever perceive in them the leaſt Shadow of a Demonſtration. I acknowledge there is ſomething in the Nature of Infinites, that ſeems not to be adequately comprehended by the human Intellect; and therefore it is no wonder, if ſome things follow from it, which the Mind of Man, involved in thick Darkneſs, is not able to conceive: and eſpecially in our preſent Queſtion, there are many things that may ſeem as Paradoxes, and incredible, to ſuch Philoſophers, who are leſs converſant in theſe Matters. However, nothing thence follows, that implies either a Contradiction, or is repugnant to any Axiom or Demonſtration. But let us conſider the Cavils produced by the Atomical Philoſophers. The firſt is that of *Epicurus:* If Quantity was diviſible *in infinitum*, it would contain an infinite Number of Parts, and ſo a Finite would contain an Infinite, which is abſurd. But I deſire they would explain their Terms, and tell us what they mean by theſe words, *An Infinite cannot be contained by a Finite.* If they ſay an infinite Magnitude cannot be contained in a finite Magnitude, I acknowledge it; but the contrary to this does not follow from our Doctrine, nor can they thence ever deduce it by a neceſſary Conſequence. If they ſay that an infinite Number of Parts, and thoſe infinitely ſmall, cannot be contained in a finite Magnitude, this is the very thing they ought to make out. They would not, I ſuppoſe, have us believe their Aſſertions without any Proof; nor would they place amongſt the Axioms, as a ſelf-evident Propoſition, ſuch a one, as we have demonſtrated by ſo many ſtrong Reaſons the contrary to be true. They may therefore urge, that an infinite Number of Parts muſt compoſe an infinite Magnitude: but this is again to beg the queſtion; for that is the thing itſelf about which we diſpute, *viz.* Whether a finite Magnitude can have an infinite Number of Parts? It is certain, whatever Number of Parts it has, whether finite or infinite, they are equal to their Whole; for as ten tenth Parts of an Unit make an Unit, an hundred hundredth Parts of an Unit taken together do likewiſe compoſe an Unit, and a thouſand thouſandth Parts collected in one Sum cannot be greater than the Whole; ſo likewiſe the infinite infiniteſimal Parts of any Magnitude are equal to that Magnitude. Or thus: Let the right Line A B be divided into an hundred Parts; all theſe taken



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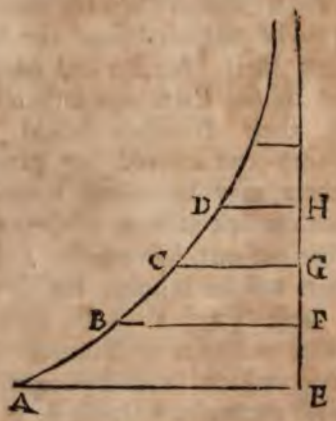
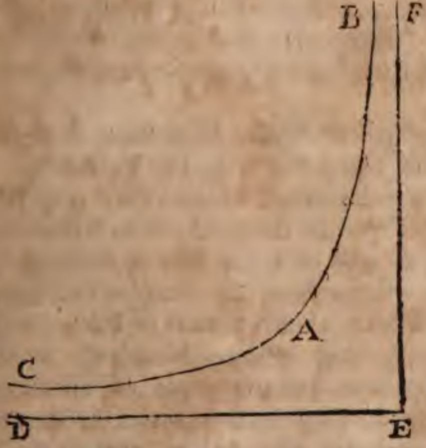
together will be equal to A B. And after the ſame manner, if the right Line A B be ſuppoſed to be divided into a thouſand Parts, theſe thouſand Parts taken together will compoſe a Magnitude neither greater or leſs than the right Line A B. Or again, if the right Line A B be divided into a million of Parts, theſe taken together will be equal to A B the Whole. And univerſally, if there be taken two Magnitudes A B and C, ſo that C ſhall have the ſame Ratio to A B, as a Unit has to any Number N, the Quantity C multiplied by the Number N will be equal to A B. For ſince the Quantities C, A B, Unity and the Number N are Proportionals, the Extremes and the Means multiplied into each other will be equal; but ſince A B multiplied by a Unit is equal to A B, (for a Unit neither increaſes by Multiplication or diminiſhes by Diviſion) the Quantity C multiplied by the Number N, will be equal to A B: therefore however great or ſmall the Number N is, this multiplying the Quantity C will always make a Product equal to A B, if the Quantity C be ſo taken, that it has the ſame Proportion to A B, as Unity has to the Number N. And therefore if N is an infinite Number, and C the infiniteſimal Part of the right Line A B, that is, the Quantity C has the ſame Ratio to A B, as a Unit has to an infinite Number N; alſo the Quantity C multiplied by an infinite Number N, that is, taken an infinite number of times, will be equal to the Quantity A B, nor as it can be greater, neither can it be leſs. If therefore the Magnitude of the Parts is diminiſhed in the ſame Ratio as their Number is increaſed, the Whole made up of all theſe Parts will remain the ſame: nor ought any Quantity to be meaſured from the Number of its Parts, but from their Number and Magnitude conjointly; ſo that if the Parts are infinitely ſmall, it is neceſſary that their Number ſhould be infinitely great, before they can exceed any given Quantity. But beſides, there may be many Examples brought both from Arithmetick as well as Geometry, where, by the confeſſion of our Adverſaries themſelves, the Number of Parts will be infinite, but the Magnitude compoſed of thoſe Parts ſhall be finite. Let the firſt Example be a Series of Numbers decreaſing in any proportion, which ſhall be equal to a finite Number, as, for example, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, &c. the Sum of this Series continued *in infinitum* will be equal to a Unit; but ſince the Series is continued *in infinitum*, its Terms will be infinite in Number: wherefore in this caſe the Parts of a Quantity, that are infinite in Number, will make a finite Quantity. And in like manner the Sum of this Series, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, &c. when continued *in infinitum*, will be equal to half an Unit, as may be demonſtrated by Arithmetick. But no body will deny, that this Series, being continued *in infinitum*, has an infinite Number of Parts; wherefore there may be an infinite Number of Parts of a Quantity, which however ſhall not exceed one half of an Unit. And in Geometry it is known that there may be given a Space infinitely long, which however ſhall be exactly equal to a finite Space; and this in an infinite number of Examples has been demonſtrated by the famous Geometers, *Torricellius*, *Wallis*, *Barrow*, and others; from whom we ſhall produce a few Inſtances. And firſt of all, let the Curve A B C D be of ſuch a nature, that if there are taken in the Aſymptote E H the equal right Lines E F, F G, G H, or the right Lines E F, E G, E H are ſuppoſed in Arithmetical Proportion, and to the Points E, F, G, H, are applied the Ordinates A E, B F, C G, D H, which Ordinates let be in a Geometrical Proportion: the Curve A B C D is call'd the Logarithmetical Curve, and the endleſs Space contained betwixt the Aſymptote and the Curve infinitely produced, will be equal to a finite Space, as is demonſtrated by the Great Dr. *Barrow* in his *Geometrical Lectures*. Whence may be ſhewn the abovementioned Property of Numbers decreaſing in any Geo-



Geometrical Proportion. But that we may apply his to our preſent Purpoſe; nobody will deny that there are, in the boundleſs Space H G F E A B C D, which is infinitely long, an infinite Number of Parts, but the Geometers demonſtrate all thoſe Parts to be equal to a finite Space; wherefore there are ſome Parts of Space, tho' infinite in Number, yet they do not make an infinite, but a finite Space. After the ſame manner, in all Hyperbolas, except that of *Apollonius*, the Area contained betwixt the Curve and the Aſymptote infinitely produced, will be perfectly quadrable, and equal to a finite Area; but in all theſe Areas there are an infinite Number of Parts, wherefore an infinite Number of Parts will be equal to a finite Quantity. Beſides, in the *Apollonian* Hyperbola C A B, although the indeterminable Area contained betwixt the Curve A B and its Aſymptote E F produced *in infinitum*, is an infinite Area, or



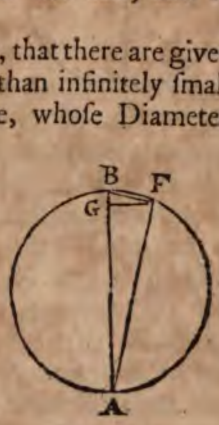
greater than any finite one; yet if that infinite Area revolves about its Aſymptote, it will thereby generate a Solid, or Body truly infinitely long, which however will be equal to a finite Solid or Body; as has been moſt elegantly demonſtrated by *Torricellius*, who called this Solid, *Hyperbolicum acutum:* but in this Solid there are an infinite Number of Parts, ſince it is infinitely long; therefore the Parts of a Body infinite in Number, may compoſe a finite Body. We might produce innumerable other Examples of this, but we have perhaps beſtowed too much time in anſwering this Objection.

SECONDLY, The Atomiſts object, If all Quantity is diviſible *in infinitum*, any the leaſt Magnitude will be equal to the greateſt, ſince the leaſt has as many Parts as the greateſt. But what conſequence, I pray, is this? Becauſe a Yard may be divided into an hundred Parts, and a Foot may be likewiſe divided into the ſame number of Parts, therefore does it follow that a Foot is equal to a Yard? But there can be nothing found more alike, than this Way of Argumentation and the preſent Objection; which is grounded on a moſt falſe Hypotheſis, which ſuppoſes that Magnitudes are to be meaſured by their number of Parts alone, without any regard to their Quantity.

THEY farther object, If a Foot may be divided into infinite equal Parts, and a Yard may be ſo divided, that every Part of the Yard may be equal to one of the Parts of the Foot, the Number of Parts in the Yard will be triple the Number of Parts in the Foot; whence ſince the Number of Parts in the Foot is infinite, the Number of Parts in the Yard will be triple that infinite Number, whence there will be given an Infinite triple of another Infinite. But whence do they learn that this is an Abſurdity? Does it contradict any Axiom commonly received? Not at all, for there is no Axiom that ſuppoſes all Infinites equal. Nor is it contrary to the Nature of an Infinite, that there ſhould be another Infinite ſtill greater: for if there be given an Infinite, as, for example, an infinite Line, there will be in it an infinite Number of Miles, more Furlongs, and ſtill more Feet. And ſo in Space, which we ſuppoſe to be infinitely extended, if two parallel Lines be produced *in infinitum*, the Area contained under theſe two Lines will be really an infinite Area, for it will exceed every finite Area, or ſuch as is bounded on all ſides; it will therefore contain an infinite Number of Acres, more ſquare Perch, and ſtill more ſquare Feet. Again, if betwixt theſe Lines be drawn another right Line parallel to either of them, this Line will divide the former Area into two other infinite Areas; which taken together, will be equal to the former infinite Area. It is not therefore repugnant to the Nature of Infinity, that one Infinite may exceed, be multiplied, or divided by another Infinite; theſe, I ſay, are no ways repugnant, but eaſily follow from the Nature of the Thing; and there is nobody that acknowledges an infinite Space, but at the ſame time he muſt be forced to own the Diviſibility of that infinite Space into others that are alſo infinite.

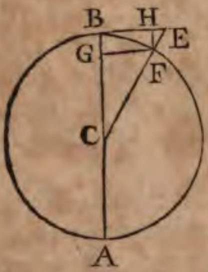
ANOTHER Argument againſt the infinite Diviſibility of Matter, is fetched from the Divine Omnipotence. God, they ſay, can reſolve any Quantity into its infiniteſimal Parts, and ſeparate thoſe Parts from one another: but if ſo, then may be given the ultimate part, and the Diviſibility would be exhauſted, therefore Quantity is not diviſible *in infinitum*. I anſwer, without doubt God is able to do whatever is poſſible, or what is not repugnant to his immutable Nature: but ſince we have already demonſtrated that there cannot be given any Particle of Matter, however ſmall, which may not be ſtill divided into other infinite Particles; it is thence manifeſt, that God cannot ſo divide Matter, as that there ſhall be given its ultimate Indiviſible. For if the Divine Power extended thus far, then God could do ſomething that involved a Contradiction, or that was repugnant to his immutable Eſſence. But they farther urge, If all Quantity is diviſible *in infinitum*, and the Parts are actually in the Extenſion, there will be actually given a Part infinitely ſmall, and conſequently not farther diviſible. In the firſt place, I anſwer, It may be denied with *Ariſtotle* that the Parts are actually in the Extenſion, and then their Argument falls to the ground, which they boaſted of as an unanſwerable Demonſtration. Secondly, we grant them that theſe Parts are actually in the Extenſion; we grant that there are Parts infinitely ſmall and indiviſible; laſtly, we grant the Argument: and yet nothing thence follows againſt the continual and infinite Diviſibility of a Quantity that is not infinitely ſmall. This is ſuppoſed indeed in the Argument, but without any Proof. Does it, becauſe an infinite ſmall Part of any Extenſion is not farther diviſible, therefore follow, that a given Part, or one not infinitely ſmall, is alſo not farther diviſible? If any thing thence follows, it is, that all continued Quantity may be reſolved into infinitely ſmall Parts, and therefore it is infinitely diviſible. But the third and true Anſwer is, by denying that the Parts in the Extenſion are ſo minute or ſmall, that they cannot be farther diviſible; and tho' there are given Parts infinitely ſmall, or ſuch as have the ſame proportion to their Whole, as a finite to an infinite Number, or a finite Space to an infinite one; yet we deny that theſe Parts are not farther diviſible: but ſince they are extended, they will be alſo diviſible, not only in two or three, or more Parts, but likewiſe every one may be divided *in infinitum*. The infinite Number of Parts of an infinitely ſmall Quantity, are wont to be called by the Geometers, Infiniteſimals of Infiniteſimals, or Fluxions of Fluxions; and theſe are uſed by them in the ſolving of many intricate Problems. Beſides, there are given other Fluxions of theſe Fluxions, or Parts that are infinitely leſs than their Wholes; and again, there are other Parts of theſe Parts, and ſo on at pleaſure. I do not deny, but, from the Weakneſs of the Human Underſtanding, this is very difficult to conceive: however, the Truth that is ſupported by ſuch powerful Arguments is not to be deſerted, eſpecially ſince there are ſome things, which we do moſt certainly know, which yet are very difficultly received by our weak Underſtanding. We might produce many Arguments, but we ſhall only bring ſuch as ſerve to illuſtrate our preſent Purpoſe; whereby we ſhall ſhew, that there are Quantities infinitely leſs than ſome Quantities, and yet alſo infinitely greater than others: ſo, if there are given infinitely ſmall Quantities, there will be ſome Quantities infinitely leſs than theſe: and again, there may be others infinitely leſs than the laſt, and ſo always on *in infinitum*.

FIRST then, we ſhall thus prove, that there are given Quantities that are infinitely leſs than infinitely ſmall Quantities. Let A B F be a Circle, whoſe Diameter is A B; and let B F be an infinitely ſmall Part of its Circumference, whoſe Chord therefore ſhall be alſo infinitely ſmall, that is, the Chord B F ſhall have to any determinate Magnitude, as, for example, to A B the Diameter of the Circle, the ſame proportion, as any finite Magnitude has to an infinite one. Let fall from the Point F, on A B, the Perpendicular F G; B G will be infinitely leſs than the right Line B F. For let be drawn A F, and the Angle A F B in the Semicircle will be a right one. And therefore in the

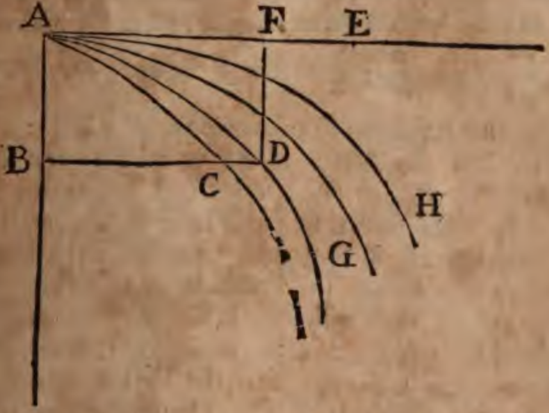


the Triangle A F B rectangular at F, from the Perpendicular F G being let fall on the Baſe A B, it will be, by the eighth of the ſixth Book of *Euclid*'s Elements, A B to B F as B F to B G; but, by Hypotheſis, A B is infinitely greater than B F, wherefore B F will be infinitely greater than B G. There may therefore be a Quantity, which, although it is infinitely leſs than a given Quantity, will be infinitely greater than another Quantity.

So likewiſe, it is known in the Circle, that as the Sine of any Arch is leſs, ſo the Tangent is greater than its correſpondent Arch; and therefore the Tangent is greater than the Sine of the ſame Arch. Let therefore in the Circle, whoſe Center is C, and Diameter A B, B F be an infinitely ſmall Arch, whoſe Tangent let be B E, right Sine G F, and verſed one G B; through F draw F H parallel to A B, then H E will be equal to the Difference of the right Sine F G and the Tangent B E, which, from what we have ſhewn, is not altogether nothing. Now in the Triangles C B E, F H E, equi-angular, by reaſon of the right Angles at H and B, and the common one E, it will be, by the fourth of the ſixth of *Euclid*, C B to B E as F H to H E; but by Hypotheſis, C B is infinitely greater than B E, wherefore F H will be infinitely greater than H E: that is, in the preſent Caſe, B G the verſed Sine of an infinitely ſmall Arch, is infinitely greater than the Difference betwixt the right Sine and the Tangent of the ſame Arch. Since therefore C B is infinitely greater than B E; and B E, as we have before demonſtrated, is infinitely greater than B G; and again, by what we have now ſhewn, B G is infinitely greater than H E; the Propoſition is manifeſt.



To the farther Illuſtration of this Doctrine, we ſhall produce another Example, borrowed from the Scholium of the firſt Section of the Great Sir *Iſaac Newton*'s *Principia*. Let the Curve A C be the common Parabola, its Axis A B, and A F a Tangent at the Vertex A. The Writers of Conicks demonſtrate, that as in the Circle, ſo in the Parabola, the Angle of Contact F A C is infinitely leſs than any rectilinear Angle. To the Axis A B and Vertex A, let there be ſuppoſed to be deſcribed a Parabola of another kind, as, for example, the Cubical, whoſe Ordinates increaſe in a ſubtriplicate Ratio of the Abſciſſas; the Angle of Contact F A D will be infinitely leſs than the Angle of Contact of the Parabola F A C; or, which is the ſame thing, there can be deſcribed no *Apollonian* Parabolas, or no Circles, with how great ſoever a Parameter, that will paſs betwixt the Cubical Parabola and its Tangent at the Vertex: which may be thus eaſily demonſtrated. Let the Parameter of the *Apollonian* Parabola A C be called a, the Parameter of the Cubical Parabola A D let be b; let there be taken in the Tangent the Point E in ſuch a manner, that



that AE may be a third proportional to the right Lines a and b, that is, that $a \times AE = b^2$; through any Point F betwixt A and E let be drawn FD parallel to the Axis, meeting the Curve AD in D, and let BCD be drawn parallel to the Tangent, and let BD, an Ordinate to the Parabola AD, be called z; but BC, an Ordinate to the Parabola AC, y; and let the Abciſſa AB be x: it will be from the Nature of theſe Curves $ax = y^2$, and $b^2 x = z^3$, and therefore $\frac{y^2}{a} = x = \frac{z^3}{b^2}$; whence $b^2 y^2 = a z^3$, and ſo by reducing this Equation to an Analogy, $b^2 : az :: z^2 : y^2$ that is, b^2 or $a \times AE$ is to az or $a \times BD$ or $a \times AF$ as BD q to BC q: but $a \times AE$ is greater than $a \times AF$, wherefore BD q will be greater than BC q, and conſequently BD greater than BC; the Point C therefore falls within the Parabola AD. The ſame is true of all the Ordinates BC, that are leſs than the right Line AE; and therefore the Portion of the *Apollonian* Parabola AC, at the Vertex falls within the Cubical Parabola. The Demonſtration is the ſame for any other *Apollonian* Parabola; conſequently there can be drawn no Parabola, and therefore no Circle (which is always of the ſame degree of Curvity with ſome Parabola) betwixt the Cubical Parabola and its Tangent at the Vertex.

How much ſoever therefore the parabolical or circular Angle of Contact is diminiſhed, it will be yet greater than the Angle of Contact at the Vertex of the cubical Parabola; and therefore any given circular or parabolical Angle of Contact, will be infinitely greater than the Angle of Contact at the Vertex of the cubical Parabola: for that Quantity is infinitely greater than another, which how much ſoever diminiſhed, it ſhall always exceed that other.

Again, to the ſame Axis and Vertex let be deſcribed another parabolical Curve AG, whoſe Ordinates always increaſe in a ſubquadruplicate Ratio of the Abſciſſas; the Angle of Contact F A G will be infinitely leſs than the Angle F A D: which is eaſily demonſtrated by a way of Argumentation not unlike the former. After the like manner, to the ſame Axis and Vertex another parabolical Curve A H may be deſcribed, whoſe Ordinates increaſe in a ſubquintuplicate Ratio of the Abſciſſas, in which the Angle of Contact F A H will be infinitely leſs than the Angle F A G: and ſo one may proceed *in infinitum*, always aſſigning more and more parabolical Figures, whoſe Angles of Contact ſhall differ infinitely from each other; that is, the Angle F A C will be infinitely leſs than any rectilinear Angle, and the Angle F A D infinitely leſs than the Angle F A C, and the Angle F A G infinitely leſs than the Angle F A D: and ſo there will be a Series of Angles of Contact proceeding *in infinitum*, whereof every latter Angle is infinitely leſs than the former; nay, between any two Angles there may be innumerable other Angles inſerted, that ſhall infinitely exceed each other. And alſo between any two of theſe Angles there may be inſerted an infinite Series of intermediate Angles, whereof each following Angle ſhall be infinitely leſs than the foregoing. Moreover, there may be innumerable Angles infinitely greater than the circular Angle of Contact, which yet ſhall be infinitely leſs than any rectilinear Angle; and ſo you may proceed *in infinitum*, nor does Nature know any limits.

I HAVE exhibited theſe Examples, that our Adverſaries may perceive, how prodigiouſly different their Speculations, concerning the Natures of Things, are from the Things themſelves.

LECTURE V.

Of the Subtility of Matter.

HAVING, as we believe, proved by undeniable Arguments, the infinite Diviſibility of Matter; and ſufficiently anſwered and refuted the Objections that ſeemed to carry any weight along with them: it remains, that we conſider a little the wonderful Subtility of Nature, and thoſe minute Particles into which Matter is actually divided, or of which it is compounded. It would be very eaſy, by an abundance of Examples, to place theſe, as it were, before your Eyes, to expoſe them to your Senſes, and even to ſhew their Smallneſs by a Calculation: but we ſhall produce only a few Inſtances.

AND firſt of all, from the great Ductility of Gold, ſeveral learned Men, as Monſ. *Rohault*, in his *Phyſicks*; our Countryman, the Honourable Mr. *Boyle*, in his Book of *Effluvia*; and lately, the famous Dr. *Halley*, in the *Philoſophical Tranſactions*, N° 194, have made a Computation of the extreme Smallneſs of its Parts. Dr. *Halley* has ſhewn, that one Grain of Gold may be cut into 10000 viſible Parts; and therefore ſince a Grain of Gold is nearly equal to $\frac{21}{100000}$ of a cubick Inch, it follows that a cubick Inch of Gold may be divided into 47 619 047 Parts, which ſhall be all diſcernable by the naked Eye.

BESIDES, Dr. *Halley* has computed the Thickneſs of that very fine Skin of Gold, with which in gilding the Artificers cover Silver-Wire; and has found it not exceed the $\frac{1}{124500}$ of an Inch: that is, if an Inch in Length was divided into 124500 Parts, the Thickneſs of the gilding would ſcarce exceed one of thoſe Parts; and therefore the Cube of the hundredth Part of an Inch, or which is the ſame thing, the $\frac{1}{1\,000\,000}$ Part of a cubick Inch would contain 243 000 000 ſuch Parts.

THAT noble Philoſopher Mr. *Robert Boyle*, has given, in his Book *of the Nature and Subtility of Effluvia*, many Experiments concerning this Subject; whence we ſhall here borrow two or three. And, firſt of all he diſſolved one Grain of Copper in Spirit of Salt *Armoniack*, and that Solution being mixed with diſtilled Water, gave a very deep and conſpicuous blue Tincture to 28534 Grains of Water: whence, ſince the Quantity of Water, whoſe Weight is one Grain, is equal to $\frac{37}{10000}$ of a cubick Inch, 28534 Grains of Water will be equal in Magnitude to 105, 57 cubick Inches. Since therefore one Grain of Copper can give a blue Colour to ſo great a quantity of Water, it is neceſſary that there muſt be ſome Part of this Copper in every viſible part of that quantity of Water; and therefore as many as are the Parts of that Water viſible to the Eye, into ſo many Parts at leaſt was that one Grain of Copper divided. But a Line is ſenſible to the Sight whoſe Length is the hundredth part of an Inch, and conſequently the Square or the Cube of that Line, will be much more diſcernable by the Sight: wherefore, ſince the Cube whoſe Side is $\frac{1}{100}$ of an Inch, is the $\frac{1}{1000000}$ part of an Inch, it follows, that at the leaſt in 105, 57 cubick Inches of Water, there are 105 570 000 Parts diſtinguiſhable by the Sight: and therefore by this Solution one Grain of Copper was at the leaſt divided into as many Parts. But the Magnitude of a Grain of Copper is equal to about $\frac{55}{100000}$ Parts of an Inch; and therefore, ſince a cubick Inch contains almoſt 20000 ſuch Parts, it hence follows, that a cubick Inch of Copper may be actually reſolved into 2 111 400 000 000 Parts: and if there be taken the leaſt Grain of Sand, ſuch a one, for example, as its Diameter may be the hundredth part of an Inch, or, which is the ſame thing, the Grain of Sand itſelf be the million part of an Inch, this will contain 2114000 ſuch Parts, into which the Copper was divided.

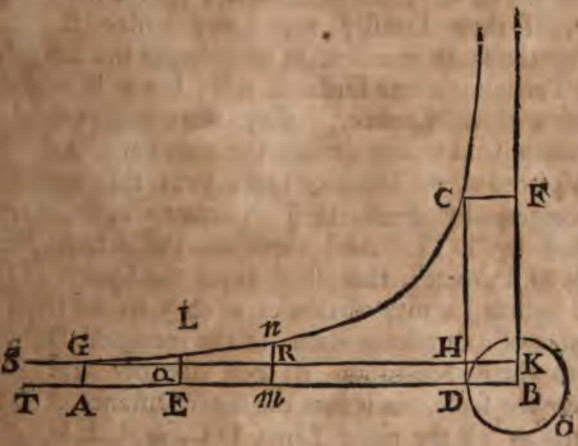
THE ſecond Example that we propoſe, ſhall be drawn from the following Principles.

ALL the modern Philoſophers agree that Odours ariſe from Effluvia, that proceeding from the odoriferous Bodies, are diſperſed on every ſide into the Medium, and which, by means of the Air we draw up our Noſtrils, do ruſh upon the olfactory Nerves, and ſo affect the Senſorium; whence it follows, that in whatever place the Odour of any Body is ſenſible, there muſt be in that very place ſome Particles of the odoriferous Body affecting the Senſe. But there are many odorous Bodies which are eaſily ſmelt at the diſtance of five feet, and which do there affect the olfactory Senſe; there muſt therefore be ſome Particles of the odorous Body diffuſed through all that Space, inſomuch that whereever our Noſtrils ſhall be placed in that Space, there, it is neceſſary, that there be ſome Effluvia of the odoriferous Body; at leaſt there muſt be ſome in that quantity of Air, that we draw through our Noſtrils in Inſpiration. Let us ſuppoſe then, that there is but one Particle of the odorous Body in every Part of that Space, which Part ſhall be equal to the fourth Part of a cubick Inch; and tho' it is probable, Effluvia ſo rare will ſcarce affect the Senſe, yet we ſhall at preſent aſſume no more; ſo many therefore at the leaſt will be the Particles producing the Odour, as there are Spaces, each of which are equal to the fourth part of an Inch, in a Sphere, whoſe Semidiameter ſhall be five feet; but in that Sphere there will be 57839616 ſuch Spaces: ſo many therefore will be the Particles producing the Odour in that Space.

HAVING in ſome meaſure determined the Number of Effluvia, we ſhall proceed to diſcover their Magnitude. Since it is neceſſary, that a Body muſt be diminiſhed in Weight, in proportion to the Effluvia that flow from it; the Weight of all the Effluvia proceeding from an odoriferous Body in a given time, will equal the Weight of the Part of the Body that is loſt in that time. Now Mr. *Boyle* has proved by an Experiment, that a certain Maſs of *Aſſa fœtida* expoſed to the open Air, has loſt in ſix days time the eighth part of a Grain in Weight: but ſince the Flux of Effluvia from an odoriferous Body is continual, it is manifeſt that it ought to be proportionable to the Time; and therefore in one Minute's Time the Weight of the Effluvia flowing from the *Aſſa Fœtida*, will be equal to $\frac{1}{69120}$ of a Grain. But the Magnitude of a Particle of Water, whoſe Weight is one Grain, is equal to $\frac{369}{100000}$ Parts of an Inch; and therefore a Particle of the ſame Water, whoſe Weight is $\frac{1}{69120}$ of a Grain, will be equal in Magnitude to $\frac{533}{10000000000}$ of a cubick Inch: but the Gravity of *Aſſa fœtida* to that of Water is (as I myſelf have experienced) as 8 to 7, and therefore the Magnitude of a Quantity of *Aſſa fœtida*, whoſe Weight is $\frac{1}{69120}$ of a Grain, will be equal to $\frac{466}{10000000000}$ of a cubick Inch. But the Number of all the Effluvia was found before to be 57839616, and therefore ſince all theſe Effluvia are only equal to $\frac{466}{10000000000}$ Parts of a cubick Inch, each Particle will be equal to $\frac{466}{578396160000000000}$ Parts of a cubick Inch; or by reducing this Fraction to a Decimal, the Magnitude of each Particle will be equal to $\frac{8}{10\,000\,000\,000\,000\,000}$ Parts of a cubick Inch.

WE have hitherto ſuppoſed the Particles producing the Odour to be every where equally diffuſed throughout the above mentioned Diſtance; but ſince towards the Center or odoriferous Body, whence they proceed, they are more and denſer than towards the outward Superficies of the Sphere, there will be many more Particles than we have before demonſtrated. For ſince Odours (as all other Qualities, that are propagated from a Center in right Lines) decreaſe in a duplicate Ratio of the Diſtance increaſed from that Center, the Number of Particles producing the Odour, and included in a given Space, as, for example, the fourth Part of a cubick Inch, will be, at the diſtance of one foot, quadruple the Number of Particles placed in the ſame Space, at the diſtance of two feet from the Center; and nine times greater than the Number of Particles at the diſtance of three feet, and ſo of the reſt: but if they were in no place in greater number than at the extreme Superficies, the Number would then be, as before diſcovered, 57839616. It is therefore manifeſt, that their Number is far greater than the aforeſaid Number.

THAT therefore, in this caſe, we may determine the Number of the Particles producing the Odour, we ought to know the Quantity of *Aſſa fœtida* that Mr. *Boyle* expoſed to the Air; but we cannot learn this from his Writings. It is therefore neceſſary we aſſume ſome Quantity; but the leſs we make it, the greater will be, *cæteris paribus*, the Proportion of the Number of Particles proceeding from it, to the Number before found out. That therefore the Number we ſhall diſcover may not exceed the Truth, we ought to aſſume ſuch a Quantity, as probably will be greater than that which Mr. *Boyle* expoſed to the Air: and let it be equal to a Sphere, whoſe Diameter is ſix Inches, here repreſented by the Circle D H O; and let the right Line A D be five feet or ſixty Inches; A B will be 63 Inches. To the Point A on A B let be erected the Perpendicular A G,



which let repreſent the Denſity or Number of Particles in a given Space at the diſtance A B; and if at all diſtances the Denſity of the Particles was the ſame, their Number might be expounded by the innumerable right Lines E Q, *m* R, D H, &c. compleating the Parallelogram A H; that is, by the Parallelogram A H itſelf. But ſince the Number of Particles, in approaching to the Center, is ſuppoſed to increaſe in a duplicate Ratio of the diminiſhed Diſtance; at the Points E, *m*, D, and innumerable others taken in the right Line A B, let be erected the Perpendiculars E L, *m n*, D C, which let be to A G as the Square of the right Line A B to the Squares of the right Lines E B, *m* B, D B, *&c.* reſpectively; and through the Points G, L, *n*, C, and innumerable others determined after the ſame manner, let be drawn a Curve Line. Now if A G repreſents the Number of Particles at the Diſtance A B, E L will repreſent their Number at the Diſtance E B, on a Suppoſition that the Denſity of the Particles are reciprocally in a duplicate Ratio of their Diſtances from the Center; but E Q would denote their Number, if their Denſity was every where the ſame. After the ſame manner, *m n* expounds the Denſity of the Particles at the Diſtance *m* B; but *m* R would repreſent their Number, if they were uniformly denſe. So likewiſe D C will denote the Number of the Particles placed at the Diſtance D B; but if they were in all places equally denſe, their Number would be repreſented by D H. And therefore the whole Multitude of Particles, that flow from the Sphere D B O, and whoſe Denſity decreaſes as they recede from the Center of a duplicate Ratio of the increaſed Diſtance, is to their Multitude, if their Denſity was every where the ſame, as it is at the outer Diſtance A D five Feet, as all the right Lines D C, *m n*, E L, A G, to the right Lines D H, *m* R, E Q, A G; that is, as the mixtilineal Area A D C G, to the Area of the Rectangle G A D H.

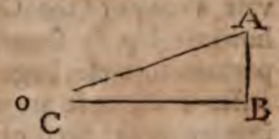
THE matter therefore is brought to this, that we inquire out the Proportion which the Area G A D C has to the Area of the Rectangle A H. But ſince the Curve G L *n* C is of ſuch a nature, that the right

Lines A G, E L, *m n*, D C, ordinately apply'd to the Aſymptote A B, are reciprocally as the Squares of the Diſtances from the Center; the Curve will be of the hyperbolick kind, and the indeterminable Space C F B T S compounded of Elements that are *reciprocal Secondanes:* and therefore that Space, altho' indeterminable, will be perfectly quadruple, and equal to twice the Rectangle C B, by what is demonſtrated by Dr. *Wallis* in his *Arithmetick of Infinites.* And conſequently the indeterminable or indefinitely extended Area C D T S will be equal to the Rectangle C B; and in like manner the indefinitely extended Area G A T S will be equal to the Rectangle G B. The Exceſs therefore, whereby the Area C D T S exceeds the Area G A T S, will be equal to the Exceſs, whereby the Parallelogram C B exceeds the Parallelogram G B. Let us therefore inveſtigate the Difference of theſe Rectangles: Since by Hypotheſis A D is 60 Inches, and B D 3, A B will be 63 Inches; and let A G be Unity, and ſince it is as D B *q* to A B *q* ſo A G to C D, that is, as 9 to 3969; C D will be 441 ſuch Parts, whereof A G is 1: and therefore C D × D B, or the Rectangle C B will be to the Rectangle B G, as 1323 to 63; and ſo the Difference of the Rectangles, that is, the Area G H D C will be 1260 ſuch Parts, whereof the Rectangle A H is 60. And therefore the Number of Particles proceeding from the *Aſſa fœtida*, whoſe Denſities decreaſe in a duplicate Ratio of the increaſed Diſtances, and contained within a Sphere whoſe Diameter is five Feet, is to their Number, (if their Denſity was every where equal to what it is at the Diſtance of five Feet) as 1260 to 60; that is, as 21 to 1. If therefore the above-diſcovered Number 57839616 be multiplied by 21, the Product will give the Number of Particles proceeding from the *Aſſa fœtida*, viz. 1214631936. Beſides, if the Fraction

$\frac{8}{10\,000\,000\,000\,000\,000}$, which expreſſed the Magnitude of the Particles in the former caſe, be divided by 21, the Quotient $\frac{8}{210\,000\,000\,000\,000\,000}$ or $\frac{38}{1\,000\,000\,000\,000\,000\,000}$ will exhibite the true Magnitude of each Particle in this ſecond Caſe.

ALL theſe things follow from hence, that we ſuppoſe a Man able to ſmell *Aſſa fœtida* at the diſtance of five feet; but there are other Animals, whoſe Senſe of ſmelling far exceeds that of a Man, as Hounds, who perceive the Effluvia of wild Beaſts remaining on the ground, a great while after the Beaſt has left the place; and ſome Birds, that get the Scent of Gun-powder at a great diſtance. The Subtility of theſe Effluvia muſt certainly be much greater than that, which we have diſcovered by the foregoing Calculation; but for want of Experiments we cannot eaſily reduce it to numbers.

THE Philoſophers, that they may ſtill farther ſhew the Subtility of Matter, produce the Example of thoſe *Animalcula*, that are obſerved floating in various Liquors, and the Seed of Animals; theſe indeed are ſo ſmall in ſome Fluids, that they appear like Points, through Microſcopes that magnify the Object very much. And that curious Obſerver of Nature, Mr. *Lewenhoeck* has diſcovered more Animalcules in the Milt of one Cod, even than there are Men at preſent living on the face of the Earth. But let us diſcover the Magnitude of theſe Animalcules: towards which, we ſhall borrow from Opticks what follow. Firſt, the Image of any Object appears under the ſame Angle, at the Vertex of Emerſion of the Lens, as the Object itſelf appears under at the Vertex of Incidence; this is demonſtrated by Dr. *Gregory*, in the 18th Propoſition of his *Elements of Dioptricks*. Secondly, it is found by experience, that thoſe Objects that appear as Points, that is, whoſe Parts the Sight is not able to diſtinguiſh, are ſeen under an Angle not exceeding a Minute. Thirdly, it is certain by frequent Obſervation, that moſt of thoſe Animalcules are of ſo ſmall a Magnitude, that they appear thro' a Lens, whoſe focal Diſtance is the tenth part of an Inch, as ſo many Points; that is, their Parts cannot be diſtinguiſhed; and therefore they appear from the Vertex of that Lens, under an Angle not exceeding a Minute. We are now to inveſtigate the Magnitude of an Object that appears under a given Angle at a given Diſtance: In the preſent Caſe, let C be the Vertex of the Lens, A B the Length of the Animalcule, B C its diſtance from the Lens equal to $\frac{1}{10}$ of an Inch, and the Angle B C A under which it is ſeen at that Diſtance equal to one Minute: from B C and the Angle B C A being given, we are to find A B the Length of the Object. Now in the rectangular Triangle A B C, there being given (beſides the right Angle at B) the Angle B C A one Minute, and the Side B C equal to the tenth part of an Inch, by Trigonometry the Side A B will be found nearly equal to $\frac{3}{100\,000}$ of an Inch. If therefore theſe Animalcules were of a cubick Figure, that is, of the ſame Length, Breadth and Thickneſs, their Magnitude would be expreſſed by the Cube of the Fraction $\frac{3}{100\,000}$; that is, by the Number $\frac{27}{1000\,000\,000\,000\,000}$; that is, to ſo many parts of an Inch is each Animalcule equal.



HENCE, what ſome Philoſophers have dreamed concerning Angels, is true of theſe Animalcules, *viz.*

that many thouſand of them may dance on the Point of a ſmall Needle.

HENCE alſo we may gather, how vaſt the Diſtance is betwixt the ſmalleſt floating Animals and the largeſt ſort, how little Proportion the firſt have to the huge Whales, that appear in the Ocean like Mountains, as oft as they raiſe their Heads above the Water. For there are in ſome Liquors Animalcules ſo ſmall, as upon a Calculation, the whole Magnitude of the Earth is not large enough to be a third proportional to thoſe minute floating Animals and the vaſt Whales of the Ocean; ſo that the Earth itſelf, however big it ſeems, is found to bear a leſs proportion to theſe large Fiſhes, than they do to thoſe leſſer ones, that are beheld through a Microſcope, floating in the Seed of Animals.

SINCE every Animalcule is an organized Body, let us a little conſider, how delicate and ſubtile ought the Parts to be, that are neceſſary to conſtitute it, and to preſerve its vital Actions. Certainly it is not eaſy to conceive, how it is poſſible there ſhould be contained in ſo narrow a compaſs, the Heart that is the Fountain of its Life, the Muſcles neceſſary to its Motions, the Glands for the Secretion of its Fluids, the Stomach and Bowels to digeſt its Food, and other innumerable Members, without which it is impoſſible an Animal ſhould ſubſiſt. But ſince every one of theſe Members is alſo an organical Body, they muſt have likewiſe Parts neceſſary to their Actions. For they conſiſt of Fibres, Membranes, Coats, Veins, Arteries, Nerves, and an almoſt infinite Number of fine Tubes like to theſe, whoſe Smallneſs ſeems to exceed the very Force of the Imagination. But there are ſome Parts that ought to be almoſt infinitely leſs than theſe, as the Fluids that flow along theſe fine Tubes: and ſuch are the Blood, Lymph, and Animal Spirits, whoſe Subtility even in large Animals is incredible.

LET us then conſider the moſt groſs Parts of the Blood in theſe Animalcules, to wit, the Globules that ſwim in the Blood, and let us endeavour to reduce their Magnitude to a Calculation.

TOWARDS the effecting of which, we ſhall make uſe of the following Hypotheſis; *viz.* That the ſimilar ſolid Parts of different Animals, that is, the ſimilar corporeal Particles, or the Parts conſiſting of three Dimenſions, are as the Magnitudes of the reſpective Animals. Whence it follows, that the ſimilar linear Dimenſions of different Animals, are in a ſubtriplicate Ratio of the Magnitudes of the Animals; that is, as the cubick Roots of theſe Animals: as, for example, the Heart of a Man is to the Heart of any Animalcule ſeen through a Microſcope, as the Body of that Man is to the Body of the Animalcule; and therefore, if the Hearts of both are ſimilar Bodies, the Diameter of the one will be to the Diameter of the other, as the cubick Root of the Magnitude of one, to the cubick Root of the other's Magnitude. So likewiſe the ſmalleſt Blood-Veſſels in a Man, are to the like ſmalleſt Veſſels in an Animalcule, as the Magnitude of the Man to the Magnitude of that Animalcule; and the Diameter of a capillary Veſſel in the Body of a Man, is to the Diameter of the like Capillary Veſſel in the Body of an Animalcule, as the cubick Root of the Magnitude of the Man to the cubick Root of the Magnitude of the Animalcule.

LET us now ſuppoſe the mean Magnitude of a Man to be three cubick Feet, or 5184 Inches; as therefore the Magnitude of a middle-ſized Man, or 5184 cubick Inches, is to the Magnitude of the Animalcule, found out above, *viz.* $\frac{27}{1\,000\,000\,000\,000\,000}$ Parts of a cubick Inch, ſo are the capillary Veſſels in a human Body to the like capillary Veſſels in an

Animalcule; and as the cubick Root of the Magnitude of a Man, or the cubick Root of the Number 5184 to the cubick Root of the Magnitude of an Animal, or to the cubick Root of the Number $\frac{27}{1\ 000\ 000\ 000\ 000\ 000}$, that is, nearly as 17 to $\frac{3}{100\ 000}$, ſo is the Diameter of a capillary Veſſel in a human Body to the Diameter of a capillary Veſſel in an Animalcule. But Mr. *Lewenhoeck* has by the help of a Microſcope diſcovered Veſſels in a human Body ſo ſmall, that if the Diameter of a Grain of Sand be ſuppoſed equal to $\frac{1}{30}$ of an Inch, it will contain 2640 Diameters of the Veſſels which he found out in the Body of a Man; and therefore the Diameter of one of theſe ſmall Veſſels will be equal to $\frac{1}{2640}\times\frac{1}{30}$ of an Inch, that is, equal to $\frac{1}{79200}$ Parts of an Inch. And although it is certain theſe Veſſels were not the leaſt of all that are in a human Body, for that there muſt be others much leſs than theſe, is eaſy to be ſhewn; but however we ſhall ſuppoſe theſe to be the leaſt of all. Let it be therefore as 17 to $\frac{3}{100\ 000}$, ſo $\frac{1}{79200}$ to another Number; which Number will expreſs in the parts of an Inch, the Diameter of the ſmalleſt Veſſel in an Animalcule: which working by the Rule of Three, is found to be $\frac{3}{134\ 640\ 000\ 000}$, this Fraction reduced to a Decimal, will be nearly $\frac{22}{1\ 000\ 000\ 000\ 000}$ or in round Numbers) $\frac{2}{100\ 000\ 000\ 000}$. But ſince it is neceſſary that the Diameter of the Globule, or the fluid Particle, that is contained in any Veſſel, be not greater than the Diameter of that Veſſel; the Diameter of a Globule of Blood that flows through theſe ſmalleſt Veſſels, will not be greater than $\frac{2}{100\,000\,000\,000}$ Parts of an Inch. And therefore the Solidity or Magnitude of theſe Globules will be leſs than the Cube of that Diameter, that is, leſs than $\frac{1}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$ Parts of a cubick Inch: that is, the Magnitude of a Globule is leſs than that part of a cubick Inch, which is expreſſed by a Fraction, whoſe Numerator is eight, but its Denominator a Number conſiſting of a Unit with thirty and three Cyphers after it.

SINCE the Fraction, that expreſſes the Magnitude of theſe Globules, conſiſts of ſo many Cyphers, that their true Quantity cannot thence be readily conceived; we ſhall proceed farther, and compare theſe Globules with ſuch other ſmall Bodies as are viſible to the naked Eye, *viz.* with the ſmalleſt Grains of Sand, ſuch, for example, as their Diameters do not exceed the hundredth part of an inch; and laſtly, we ſhall compare theſe ſmall Grains of Sand with other great Bodies of the Earth, as, for example, huge Mountains; that we may perceive what proportion they bear to one another: and ſo we ſhall the more eaſily comprehend the Smallneſs of Particles. But why do I make uſe of this word? ſince I ſhould rather ſay, by this Compariſon their Subtility will appear incomprehenſible. For it may be gathered from thence, that Ten thouſand two hundred and fifty ſix of the higheſt Mountains in the whole Earth do not contain as many Grains of Sand, as one Grain of Sand can of the Blood-Globules of theſe Animalcules. It is no wonder, if you here ſtand amazed, and being ſtruck with ſo prodigious a thing, ſhould call in queſtion the infinite Diviſibility of Matter, although it is ſupported by uncontroulable Demonſtrations. But however incredible this

Aſſertion may appear at firſt ſight, we ſhall notwithſtanding deduce it from eaſy and evident Principles.

THAT our Calculation may proceed the eaſier, we ſhall call the tenth part of a Foot an Inch, and ſuppoſe that if an hundred Grains of Sand were placed one by another, they would occupy the Space of an Inch in length; or, which is the ſame thing, let a thouſand contiguous Grains of Sand be ſuppoſed to be extended through the Length of one Foot, there will therefore be in one cubick Inch 1 000 000 Grains of Sand, and in a cubick Foot there will be 1 000 000 000 Grains. Let a Mile or 1000 Paces be equal to 5000 Feet, there will be then 125 000 000 000 cubick Feet in a cubick Mile: ſo that the Number of Grains of Sand, that can be contained in a cubick Mile, will be 125 000 000 000 000 000 000.

NOW that we may have the Dimenſions of the Mountains, we will make choice of the higheſt, as it is ſuppoſed, of the whole Earth, *viz.* that which is in the Iſland of *Teneriff*, and called *El Pico de Terrario*, whoſe perpendicular Altitude is vulgarly eſteemed three *Italian* Miles. We will ſuppoſe the Figure of this Mountain to be a Cone, and its Circumference at the Baſe to be five and thirty Miles, the Area of the Baſe will be about 97,5 Miles; for as 314 to 100, that is, as the Circumference of a Circle to its Diameter, ſo is 35 to 11,14 the Diameter or Thickneſs of the Mountain at its Baſe; 27,85 the fourth of which being multiplied by the Circumference 35, gives the Area of the Baſe, *viz.* 97,5 ſquare Miles. Since therefore the Mountain is, by Hypotheſis, of a conick Figure, if its Baſe be multiplied into a third part of its Height, the Product will give the ſolid Content of the Mountain; and the third part of the Height is, by Suppoſition, equal to one Mile, which multiplying the Number 97,5, the Product or Solidity of the

Mountain will be equal to 97,5 cubick Miles: which Number, if it be again multiplied by 125 000 000 000 000 000 000, the Product or Number 12 187 500 000 000 000 000 000, will exhibite the Number of Grains of Sand, of which the Mountain of the Iſland *Teneriff* might be compoſed.

THESE being found out, let us ſee how many Blood-Globules may be contained in one Grain of Sand. From what has been ſhewn before, the Magnitude of each Globule is leſs than the $\frac{8}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$ Parts of an Inch; and the Magnitude of a Grain of Sand is equal to the $\frac{1}{1\,000\,000}$ Part of an Inch: ſo that if the latter Number is divided by the former, the Quotient $\frac{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}{8\,000\,000}$, or $\frac{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}{8}$; that is,

125 000 000 000 000 000 000 000 000 is leſs than the Number of Blood-Globules that may be contained within the Magnitude of a Grain of Sand: but this Number 125 000 000 000 000 000 000 000 000 divided by 12 187 500 000 000 000 000 000, the Number of Sands that may be contained in the Mountain of the Iſland *Teneriff*, the Quotient will be greater than the Number 10256. So that one Grain of Sand may contain ten thouſand two hundred and fifty ſix times more Blood-Globules in it, than the higheſt Mountain of the whole Earth does Grains of Sand; or, which is the ſame thing, ten thouſand two hundred and fifty ſix Mountains, each of which ſhall be equal to the higheſt Mountain on the whole Earth, cannot contain in them as many Grains of Sand, as one Grain of Sand may contain in itſelf of ſanguineous Particles of Animalcules, that are by a Microſcope

ſeen floating in ſome Fluids: which was to be ſhewn. Since therefore theſe Globules are of ſo ſmall a Magnitude, what muſt we think of the Particles compoſing the Fluid in which the Globules are carried, and of the Subtility of the Animal Spirits? This, no doubt, is ſo great, as to exceed all Calculation, and even the Force of the Imagination.

THIS Subtility of Nature is wonderful beyond meaſure; but there are other Particles of Matter ſtill more ſubtile than theſe, to which if the abovementioned Globules were compared, they would not only appear as Mountains, but as vaſt Earths. I mean the Particles of Light, which are darted all around from the lucid Body, with an inexpreſſible Celerity, whoſe Subtility the human Mind will perhaps be never, unleſs when it ſhall be made perfect in the Heavens, able thoroughly to comprehend. That it is immenſe, may hence be gathered, that the Light of the ſmalleſt Candle, in a time altogether inſenſible, and without any diſcernible Diminution of the Candle, may be perceived by the Eye, at the diſtance of two Miles; whence it is neceſſary, that in every aſſignable Part of the Sphere of Activity of that Candle, the Diameter of which Sphere is greater than four Miles, and in every aſſignable Portion of Time, there are ſome Particles of that Light, which enter, or are ready to enter the Eye; which will be different in different Parts of Time. And from this ineffable Subtility of Light it is, that the Sun, although from the Beginning of its Creation, it has continually emitted Light, and that with great Swiftneſs; yet in all that time has not loſt any thing ſenſible of its Magnitude, notwithſtanding it daily, tho' inconſiderably, decreaſes in Quantity: whence, tho' after ſix thouſand Years, its Diminution is not yet become remarkable, notwithſtanding after a finite Series of Years, altho' a very protracted one, it will be wholly diſſipated. Whence it follows, that this World can neither exiſt to Eternity, nor could it have exiſted from Eternity.

FROM *the Demonſtration of the infinite Diviſibility of Matter are derived the following Theorems, relating to its Rarity, and the Tenuity of its Compoſition.*

LEMMA.

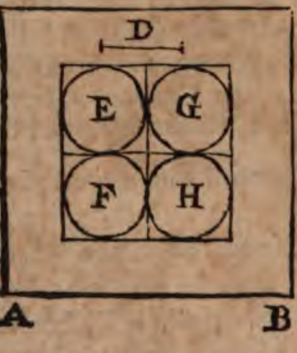
ANY Quantity of Matter being given, of it, or any part of it, a concave Sphere may be formed, whoſe Semidiameter ſhall be equal to any given right Line.

LET the Particle of Matter be a^3, and the given right Line be b. The Ratio of the Circumference of a Circle to its Radius let be as p to r. Let the Semidiameter of the Concavity be called x, the Thickneſs of the Shell encompaſſing the Spherical Concavity, will be $b-x$, and the Cylinder circumſcribed about the Spheres whoſe Radius is b, will be $\frac{p \times b^3}{r}$, whence the Sphere inſcribed within the Cylinder will be $\frac{2 \times pb^3}{3r}$; by the ſame reaſon, the Sphere whoſe Radius is x, will be $\frac{2 \times px^3}{3r}$, whoſe Difference $\frac{2p}{3 \times r} \times \overline{b^3 - x^3}$ is to be made equal to the ſpherical Shell, or given Particle of Matter: that is, it will be $\frac{2p}{3r}\overline{b^3 - x^3} = a^3$ or $b^3 - x^3 = \frac{3ra^3}{2p}$ whence $x^3 = b^3 - \frac{3ra^3}{2p}$ and $x = \sqrt[3]{\ } b^3 - \frac{3ra^3}{2p}$. So that the Thickneſs of the Spherical Shell, or $b-x$, will be $= b - \sqrt[3]{b^3 - \frac{3ra^3}{2p}}$.

BY *the ſame reaſon may be made of a given Quantity of Matter, concave Cubes, concave Cylinders, or concave Bodies of any other figure whatſoever, whoſe Sides ſhall be equal to a given right Line.*

The Firſt THEOREM.

GIVEN *any Quantity of Matter, however ſmall, and given any finite Space, however large; which, for example, let be a Cube, that ſhall circumſcribe the Orb of* Saturn: *it is poſſible that the Matter of that Grain of Sand may be diffuſed through all that Space, and ſo fill it, that there ſhall be in it no Pore, whoſe Diameter will exceed a given right Line.*



LET there be a given Cube Space, whoſe Side let be the right Line A B, equal to the Diameter of *Saturn*'s Orbit; and let be given a Particle of Matter, whoſe Quantity let be b^3, and let the given right Line (greater than which the Diameter of the Pores ought not to be) be D. Let the right Line A B be conceived to be divided into Parts equal to the right Line D, whoſe Number will be finite, when neither the right Line A B is ſuppoſed infinitely great, nor the right Line D infinitely ſmall. Let the Number be n, that is, let be $n\,D = A\,B$, ſo that it will be $n^3\,D^3$ equal to the Cube of the right Line A B. Let alſo the given Space be conceived to be divided into Cubes, each of whoſe Sides are equal to the right Line D, and the Number of the Cubes will be n^3; and theſe Cubes will be repreſented in the Figure by the Spaces E, F, G, H. Again, let the Particle b^3 be ſuppoſed to be divided into Parts, whoſe Number let be n^3, and in each cubick Space let be placed one of thoſe Particles; and by this means the Matter b^3 will be diffuſed through all that Space. Beſides, each Particle of the Matter b^3 being placed, as it were, in its Cell, may be formed into a concave Sphere, whoſe Diameter may be equal to the given right Line D; whence it will follow, that each Sphere will touch that which is next to it; and the given Particle of Matter, however ſmall b^3, will ſo fill the given Space, that there will be no Pore in it, whoſe Diameter will exceed the given right Line D. *Q. E. D.*

Cor. THERE may be a given Body, whoſe Matter if it be reduced into a Space abſolutely full, that Space may be any given Part of the former Magnitude.

The Second THEOREM.

THERE *may be two Bodies equal in Bulk, whoſe Quantities of Matter may be very unequal, and tho' they have any given Ratio to one another, yet the Sums of the Pores or empty Spaces in the Bodies, may almoſt approach to a Ratio of Equality.* Or, in the *Carteſian* Stile; *All the Space, that is poſſeſſed by a ſubtile Matter within the Pores of one Body, may be almoſt equal to the Space that is occupied by the like Matter within another Body; although the proper Matter of one Body exceeds ten thouſand or an hundred thouſand times the proper Matter of the other Body, and the Bodies may be equal in Bulk.*

LET there be, for example, a cubick Inch of Gold, and a cubick Inch of Air not condenſed. It is certain the Quantity of Matter in the Gold will exceed twenty thouſand times the Matter of Air; yet it may be, that the Spaces in the Gold, either abſolutely empty, or replete with a ſubtile Matter,

LECTURE VI.

Of Motion, Place, and Time.

SINCE we have hitherto ſufficiently treated of the Solidity, Extenſion, Diviſibility and Subtility of Bodies; we now come to explain Motion, the nobleſt Affection that Body is endued with: by the mediation whereof, Nature diſcovers herſelf acting in that Variety of Things, which ought not to be beheld without Wonder and Aſtoniſhment; and without which, all the Ornament and Beauty of the World would periſh, and a horrid Darkneſs and an infinite Numbneſs would poſſeſs every thing. On this depend the Viciſſitudes of Days and Nights, and the ſo great Variety of Cold and Heat, Snow, Rain, and Sun-ſhine ſucceeding each other, and all the Seaſons of the Year. By Motion Plants grow, Trees are nouriſhed, and Animals live; ſince Life itſelf conſiſts only in Motion, that is, the Circulation of the Blood. But why do I ſpend time in enumerating Particulars? ſince all things owe their Birth to Motion.

THE Science therefore of Motion is ſo neceſſary to philoſophize aright, that not the leaſt Operation of Nature can be inveſtigated without it. Hence the famous and moſt true Saying of the Philoſopher, Αναγκαῖον ἀγνουμένης αὐτῆς κινήσεως ἀγνοεῖσθαι ϗ τὴν Φύσιν, Motion being unknown, Nature muſt of neceſſity be ſo likewiſe.

THE Philoſophers, or rather the Metaphyſicians, have had various Diſputes concerning the Nature, Cauſes, and Communication of Motion; and the

Confuſion of Ideas, and the Darkneſs that thence aroſe, appear to have been ſo great, that amongſt the Follies of Diſputation, the natural and ſimple Notion that they had of it, ſeems to have been loſt amongſt them. For there can ſcarce be found one of the common People, or the meaneſt Artificer, but he knows more of the true Nature and Cauſe of Motion than all theſe wrangling Philoſophers together, ſome of which were arrived to that height of Folly, as to deny all Motion, as a thing impoſſible, to Bodies; and they propoſed ſome Cavils, whereby they flattered themſelves that they had demonſtrated its Impoſſibility.

We will produce here ſome of their ſtrongeſt Arguments: and the firſt ſhall be that of *Diodorus Cronus*; namely, If a Body moves, it muſt either move in the place where it is, or in the place where it is not, both which are impoſſible: for if it moves in the place where it is, it will never depart from that place, and ſo there will be no Motion; in like manner it cannot move in the place where it is not, becauſe nothing acts in a place where it is not: therefore a Body will not move at all. I anſwer, that a Body neither moves in the place where it is, nor in the place where it is not, but it moves from one place to another.

The ſecond Argument ſhall be that of *Zeno*, to which he gave the Name of *Achilles:* and thereby he endeavours to prove, that if there be ſuch a thing as Motion, *Achilles*, though ever ſo ſwift, could never overtake a Tortoiſe, though the ſloweſt of Animals. The Argument is this: Suppoſe *Achilles* to be diſtant from the Tortoiſe a certain finite Space, as, for example, a Mile, and let us ſuppoſe he moves an hundred times faſter than the Tortoiſe; therefore whilſt *Achilles* runs one Mile, the Tortoiſe has moved forwards the hundredth part of a Mile; ſo that *Achilles* has not yet overtaken the Tortoiſe: And again whilſt *Achilles* paſſes over that hundredth part of a Mile, the Tortoiſe in the mean while will have creeped through the ten thouſandth part of a Mile; inſomuch that neither has *Achilles* yet overtaken the Tortoiſe. After the ſame manner, whilſt *Achilles* runs that ten thouſandth part of a Mile, the Tortoiſe will have advanced forward the million part of a Mile; ſo that *Achilles* has not yet come up with the Tortoiſe. And ſo he may go on *in infinitum*, nor will he ever be able to overtake the Tortoiſe, but there will be always ſome diſtance betwixt *Achilles* and the Tortoiſe.

THIS is the famous Argument of *Zeno*; to anſwer which, ſome have wrote whole Treatiſes: but we ſhall eaſily diſſolve the Knot, by ſaying, That a Mile, together with the hundredth part of a Mile, together with the ten thouſandth part of a Mile, with the million part of a Mile, and ſo on *in infinitum*, will be equal to a finite Quantity. For it is demonſtrated by the Arithmeticians, that the Sum of any Series of Quantities decreaſing in any Geometrical proportion *in infinitum*, will be equal to a finite Quantity; but the $\frac{1}{100}$ part of a Mile, together with the $\frac{1}{10\,000}$ part, together with the $\frac{1}{1\,000\,000}$ part, together with the $\frac{1}{100\,000\,000}$, or the hundred million part, and ſo on *in infinitum*, is a Series of Quantities decreaſing in Geometrical Proportion *in infinitum*; ſo that its Sum, ſince it is equal to a finite Quantity, may be run over by a Body moving with a given Velocity, in a finite time. For let us ſuppoſe *Achilles* in the ſpace of an Hour to run a Mile, and therefore he will paſs over the hundredth part of a Mile in the hundredth part of an Hour, and the ten thouſandth part of a Mile in the ten thouſandth part of an Hour; and after the ſame manner he will move through the million part of a Mile in the million part of an Hour, and ſo of the reſt. If therefore one Hour, together with the hundredth part of an Hour, together with the ten thouſandth part of an Hour, together with the million part of an Hour, $+ \frac{1}{100\,000\,000}$, *&c. in infinitum*; if I ſay, the Sum of this Series continued *in infinitum* is equal to an infinite Space of Time, it is certain *Achilles* would never overtake the Tortoiſe in any finite time: but ſince, as we ſaid before, the $\frac{1}{100} + \frac{1}{10\,000} + \frac{1}{1\,000\,000}$ *&c.* Parts of an Hour, is a Series of Quantities decreaſing in Geometrical Proportion *in infinitum*; its Sum will be equal to a finite Quantity, *viz.* the ninety-ninth part of an Hour, as might be eaſily demonſtrated. And within that Space of Time, all the Particles of Time, however infinite in Number, will be paſſed by. We ſay therefore, that *Achilles* will overtake the Tortoiſe, after one Hour, and all thoſe Particles of Time contained in the foregoing Series, and which are infinite in Number, are expired; that is, he will arrive at the Tortoiſe, after one Hour and the ninety-ninth Part of an Hour: And ſo the Force of this Argument is deſtroyed, tho' its Patrons have ſo often boaſted of it as unanſwerable.

THIS Argument is alſo wont to be brought againſt Motion: Let B and C be two contiguous Points, and the Body A be moved from B to C, in an Inſtant D: when A is moved, it is ſuppoſed to be in B, and therefore in that Inſtant it cannot arrive at C, namely, becauſe it is ſuppoſed to be in B; and it cannot be in both in the ſame Inſtant, becauſe nothing can be at once in two places, that is, in the ſame Inſtant; and therefore in the Inſtant in which referred to other Bodies, and determined by their Diſtances from, and Poſitions to, other Bodies: as, for example, let us ſuppoſe any one to ſit in the Corner of ſome Houſe; his place will be defined by the Diſtance, Reſpect, and Poſition, that he has to other Corners, Walls, and ſurrounding Bodies, that are looked upon as immoveable: and as long as he keeps the ſame Diſtance and Situation from theſe Bodies, ſo long is he ſuppoſed to remain in the ſame Place. So likewiſe if any one ſits in a Ship, whether the Ship moves or not, as long as he keeps at the ſame Diſtance from all the Parts of the Ship, which are looked upon as being at reſt, and his Poſition to them all remains the ſame, his relative Place will alſo remain the ſame.

WHAT we have ſaid of Place, may be in like manner applied to Space; for that may be diſtinguiſhed into abſolute and relative. We call that abſolute, which of its own nature, and without relation to any thing beſides, always remains ſimilar and immoveable. But that is relative which is referred to ſome Bodies, by which it is determined and meaſured; whoſe Parts, to wit, always keep the ſame Poſition and Situation to thoſe Bodies, and whoſe Diſtance from them (*viz.* Bodies) always remains unchanged and the ſame.

RELATIVE Space is always the ſame with abſolute Space in Magnitude and Figure, but however it is not neceſſary that it ſhould be always numerically the ſame Space: for if we ſuppoſe the Ship to be moved, the abſolute Space contained within the Cavity of the Ship, will be different in different Places; but ſince the Cavity and Figure of the Ship remain the ſame, the Magnitude of the Space contained in it, and its Figure will be the ſame, and its Parts alike ſituated will always have the ſame Poſition and Diſtance in reſpect to the ſame Parts of the Ship: and therefore it ought to be called the ſame relative Space.

So likewiſe in the Hypotheſis of the Earth's Motion, the Space that is contained within the Walls of any Building, altho' by conſidering it as abſolute, always is changed; yet ſince the Cavity of the Building remains the ſame, there is the ſame Figure, and all the ſimilar Parts of the contained Space always keep the ſame Situation to the ſame Parts of the Building; and ſince they always obſerve the ſame Poſition to the relative Space of our Air, as alſo to all the Parts of the Earth, that Space may be called relative.

And after the ſame manner may Time be diſtinguiſhed into abſolute and relative. Abſolute Time flows equally, that is, it never proceeds faſter or ſlower, but without any relation to the Motion of Bodies, it glides along with an equal Tenour. Relative or apparent Time is the ſenſible Meaſure of any Duration by the means of Motion; for ſince the equal Flux of Time itſelf does not affect the Senſes, there muſt be called in the help of an equable Motion, as ſome ſenſible Meaſure, which may determine its Quantity, whoſe Parts may always correſpond and be proportionable to the Parts of Time. But that uniform Motion, which is to be applied to the Meaſure of Time, ought to be the moſt remarkable, obvious to every one, and affecting the Senſes of all, ſuch as are commonly thought to be the Revolutions of the Sun, and Moon, and Stars: by which we divide Time into Hours, Days, Months, and Years. And as we judge thoſe Times to be equal, that paſs whilſt a moving Body, carried by an equable Velocity, runs over equal Spaces; ſo likewiſe thoſe Times are ſaid to be equal, which flow whilſt the Sun or Moon perform their Revolutions, that to our Senſes are equal.

BUT ſince, as we ſaid before, the Flux of Time cannot be accelerated or retarded, but all Bodies are moved ſometimes faſter and ſometimes ſlower, nor perhaps is there given in nature a perfectly equable Motion; it is neceſſary that abſolute Time ſhould be ſomething truly and really diſtinct from Motion, nor does its Nature more depend on the Motion of Bodies than on their Reſt. For let us ſuppoſe the Heavens and the Stars to have remained without Motion from the very Creation of the World, it does not thence follow that the Courſe of Time would have been at a ſtand, but the Duration of that quieſcent State would be equal to the very Time that is now elapſed. Beſides, ſince we learn from the Scriptures, that in the days of *Joſhua* the Sun remained for ſome time immoveable in the ſame part of the Heavens, yet abſolute Time was not therefore at a ſtand, and began again its Courſe with the Sun, but flowed in the ſame Tenour as before, altho' all the Sun-Dials ſhewed the ſame Hour of the Day, during all the time of that Station: and ſo indeed the apparent Time, *viz.* that related to the Motion of the Sun, ſtood ſtill, when in the mean while the abſolute Time proceeded uniformly on.

So likewiſe ſince at preſent the apparent Motion of the Sun is not uniform, neither will its diurnal Revolution be equable, as all the Aſtronomers acknowledge, but ſometimes it proceeds in a ſwifter, ſometimes in a ſlower degree; and therefore the natural Day, νυχθήμερον, or the Space of Time elapſed in one diurnal Revolution, will be ſometimes leſs, ſometimes greater: ſo that the apparent Time does not proceed in the ſame Tenour as the abſolute, whence it ought to be diſtinguiſhed from it.

SINCE abſolute Time is a Quantity uniformly extended, and in its own nature moſt ſimple, it may be aptly repreſented and propoſed to our Imagination by the moſt ſimple Magnitudes: ſuch, in the firſt place, ſeem to be right Lines and Circles, with which and Time there is a certain Analogy. For all the Parts as well of Time as of right Lines and Circles, are every where ſimilar and uniform; and as a Line is generated by the Motion or Flux of a Point, whoſe Quantity depends on one Length determinated by Motion; ſo likewiſe Time may in ſome meaſure be looked upon as the Path of an Inſtant continually gliding along, whoſe Quantity flows from one Succeſſion, as it were ſtretched out in Length, and which is demonſtrated by the Length of the Space paſſed over; and therefore may be beſt repreſented by the Flux of a Point or a right Line, which will be often done in what follows.

BUT it muſt be obſerved, that we underſtand by the word Time, that Space of Time wherein Motion is performed; ſo that when we treat of philoſophical Matters and of Motion, it (*viz.* Time) may be fitly defined with *Ariſtotle*, *Menſura Motus ſecundum prius & poſterius*; that is, *the Meaſure of Motion according to what is paſt and what is to come:* not indeed as conſidering the abſolute Nature of Time, but that Connection which Motion has with it; namely, as no Space can be paſſed over in an Inſtant by a Body in Motion, but all Motion is performed ſucceſſively, and according to the Flux of Time, which Motion therefore may be compared with the Quantity of Time, and be meaſured by its Flux.

LECTURE VI.

Definitions.

I. M*OTION is a continual and ſucceſſive Change of Place.*

II. CELERITY *is an Affection of Motion, whereby a Body in Motion paſſes over a given Space in a given Time.*

III. BUT *Reſt is the Permanence of any Body in the ſame Place.*

HENCE it follows, that Reſt, Motion, and Celerity are twofold, according to the double Diſtinction of Place, *viz.* abſolute and relative.

IV. ABSOLUTE *Motion is the Change of abſolute Place, and its Celerity is meaſured by abſolute Space.*

V. ABSOLUTE *Reſt is the Permanence of a Body in the ſame abſolute Place.*

VI. RELATIVE *Motion is the Change of relative Place, whoſe Celerity is meaſured by relative Space.*

VII. BUT *relative Reſt is the Permanence of a Body in the ſame relative Place.*

FROM theſe it follows, Firſt, That a Perſon may relatively be at reſt, who yet, in reſpect to abſolute Space, may truly and abſolutely be in motion: As, for example, if any one ſhould be ſeated in a Ship, ſince he keeps the ſame relative Place, and retains the ſame Situation and Diſtance in reſpect to the other Parts of the Ship, which are looked upon as quieſcent, he is relatively at reſt; though in the mean while he is carried by the ſame Motion, with the ſame Celerity, and according to the ſame Courſe, as the Ship is borne by the Winds; in which caſe, all the Parts of the Ship keeping the ſame Situation amongſt themſelves, will appear to the Spectator placed within the Ship, as if they were at reſt: on the contrary, whilſt the Ship is in motion, the Shore and other circumjacent Bodies out of the Ship, will appear to a Spectator in the Ship, to be moved with the ſame Celerity, but towards contrary Parts, as the Ship, or it will recede from them. The Reaſon of this Appearance may be eaſily ſhewn from Opticks: for thoſe Bodies ſeem to be at reſt, which keep always the ſame Poſitions and Diſtances in reſpect to the Eye itſelf; but thoſe Bodies which we ſee to be moved, we find them change their Diſtance and Poſitions in reſpect to our Eyes. But we will conſider this matter a little farther.

SINCE Opticks teach us, that every Body, which is viſible, has, by means of the Rays which proceed from that Object, its Image painted on the bottom of the Eye, or Retina; it follows, that thoſe Objects will ſeem to be moved, whoſe Images are moved on the Retina; that is, which paſs over ſucceſſively the different Parts of the Retina, whilſt the Eye is ſuppoſed to be at reſt: but thoſe Objects will be looked upon as being at reſt, whoſe Images always occupy the ſame part of the Retina, that is, when the Motion of thoſe Images are not perceived in the bottom of the Eye. And hence it is, that they who are ſeated in a Ship, do not perceive the Motion of the Ship: for all the Parts of the Ship being relatively at reſt among themſelves, keeping the ſame Poſition and Diſtance in reſpect to the Eye, will have their Images always painted on the ſame Parts of the Retina; therefore their Motion will not be ſeen. But when the Spectator turns his Eyes towards the Shore, whilſt the Ship is moved, it is neceſſary that any external Object ſhould change its Situation in reſpect to the Eye, and therefore its Image will ſucceſſively occupy different Parts of the Retina; that is, the external Object will ſeem to move. For the ſame reaſon, if the Earth is moved about the Sun, or its own Axis, its Motion will not be perceived by the Inhabitants of the Earth; *viz.* ſince the Edifices and all viſible Objects on the Earth remaining in the ſame Parts of the Earth, will always keep the ſame Poſition amongſt themſelves and to the Eye; but if the Stars, and all other Bodies not fixed to the Earth, are beheld, thoſe, for the ſame cauſe as the Shore before, will be ſeen to move: that is, if the Earth is turned about its Axis from Weſt to Eaſt, the Sun and the other Stars will be beheld to move from the Eaſt to the Weſt.

BUT leaving for a while the Motion of the Earth, let us return to the Example of a Ship: If the Ship is carried in any Direction whatever, as, for example, towards the Eaſt; and any one ſitting in the Prow, ſhould caſt a Stone towards the Weſt, with the ſame Velocity as the Ship itſelf has towards the Eaſt; the Stone in this caſe would ſeem to the Spectator within the Ship, to be moved towards the Weſt, and its relative Velocity would be equal to the abſolute Celerity of the Ship itſelf: yet in truth the Stone would be at reſt in the abſolute Space, abſtracting from the Motion of the Earth, and all that which may ariſe from Gravity. And if we ſuppoſe any Perſon to be placed out of the Ship hanging in the Air, he would behold the Stone at reſt; but ſince the Stone is heavy, he would ſee it move only perpendicularly downwards, not tending more towards the Eaſt than towards the Weſt: for the Force impreſſed on the Stone by the Caſter, does nothing elſe but deſtroy the equal Force of Motion, that was communicated to it from the Ship moving in a contrary Direction; for any Body or Space being in motion, likewiſe all Bodies or Particles of Bodies relatively at reſt within that Body or Space, will be moved with the ſame Celerity, and in the ſame Direction.

But ſome may object, That the Stone thrown from the Hand of the Caſter, lights on the Poup, and gives it a Blow: and ſince the Stone ſtrikes againſt the Poup itſelf, it cannot but be moved. I anſwer, it is true indeed that thoſe who are within the Ship behold the Stone lighting on and ſtriking againſt the Poup; but if any one is ſuppoſed hanging in the Air out of the Ship, he would not ſee the Stone moving towards the Poup, but the Poup towards the Stone, and ſtriking it; and the Magnitude of the Stroke, that is received on each Body, will be altogether the ſame as if the Ship was at reſt, and the Stone really impelled towards the Poup with the ſame Celerity wherewith the Poup arrives at the Stone. For if there are two Bodies, A and B, however equal or unequal; the Force of Percuſſion will be the ſame, whether B with a given Celerity impinges on the Body A at reſt, or if B be at reſt, and A with the ſame Celerity ruſhes on it: or if each Body ſhould be moved towards the ſame part, and the ſubſequent Body A moving the faſteſt, ſhould impinge on B; the Quantity of the Stroke will be the ſame, as if B was altogether at reſt, and A was only born along with the Difference of Celerities, that is, whereby its Celerity exceeds the Celerity of the Body B: or laſtly, if A and B were carried towards different Parts, the Magnitude of the Stroke would be the ſame, as if one was at reſt, and the other moved, with a Celerity that is equal

to the Sum of the former Velocities. I ſay, in ſhort, the relative Velocity of Bodies always remaining the ſame, as that wherewith they arrive at one another, the Quantity of Percuſſion will be likewiſe the ſame, however the true Velocities are ſhared, as will be demonſtrated hereafter. But let us return to our Example of the Ship.

IF the Force with which a Stone is thrown by the Caſter, is leſs than that which it receives in this caſe from the Motion of the Ship, that Stone is truly carried by its abſolute Motion towards the ſame Parts, as the Ship itſelf: that is, it will be ſeen to move by the Spectator, whom we have ſuppoſed to be placed out of the Ship in the Air, towards the Eaſt, with a Celerity, whereby the Celerity of the Ship exceeds the Celerity of the Motion impreſſed by the Hand of the Caſter; but to thoſe placed in the Ship, the Stone will appear to move towards the Weſt, with the very ſame Celerity that it received from the Hand of the Caſter, with which alſo it will ſeem to ſtrike on the Poup.

BUT if any one ſitting in the Poup ſhould throw a Stone towards the Prow, its true and abſolute Motion would be towards the Prow or Eaſt, and it would be beheld by our Spectator placed out of the Ship, to be carried with a Celerity that is equal to the Sum of two Celerities, whereof one is that which the Stone receiv'd from the Caſter, the other that which was communicated to it from the Motion of the Ship.

ALL theſe things may be apply'd to the Hypotheſis of the Earth's Motion. For if the Earth only revolved about its Axis from the Weſt towards the Eaſt, and a Stone or Bullet ſhould be thrown by a Cannon to the Weſt, with that Velocity whereby the Earth is turned about its Axis; the Impetus, that the Bullet receives from the Cannon, would deſtroy the contrary Impetus, that was impreſſed on it by the Earth: ſo that the Bullet would be at reſt in the abſolute Space, if we have no regard to the Motion ariſing from its Gravity. Nevertheleſs thoſe that live on the Earth, and revolve together with it, would behold the Stone or Bullet ſwiftly carried towards the Weſt; and if any Wall was oppos'd to its apparent Motion, they would ſee the Bullet ſtriking againſt the Wall with the ſame force as if the Wall was really at reſt, and the Bullet impinged againſt it with the Celerity, which in that caſe it had received from the Exploſion: for, as we ſaid before, the Quantity of the Stroke would be the ſame, whether the Bullet was thrown with the determinate Celerity againſt the Wall at reſt, or the Wall met the Bullet at reſt with the ſame Celerity.

IF the Force that is impreſſed on the Bullet by the Exploſion of the Gun, be leſs than that which is communicated to it by the diurnal. Motion of the Earth, the Bullet will be really carried towards the Eaſt; but becauſe its Velocity is leſs than that whereby we are revolved towards the Eaſt, the Bullet will appear to us to tend towards the Weſt, and any Object that oppoſes its apparent Motion, will ſeem to be ſtruck by it with the ſame force, as if really the Obſtacle had remained in the ſame abſolute Space, and the Bullet had impinged on it with the Force it received from the Gun. If, laſtly, the Bullet be diſploded towards the Eaſt, its abſolute Motion will be towards the Eaſt, and its Velocity will as much exceed the Velocity wherewith the Earth is moved, as is that which is impreſſed on the Bullet by the Gun; ſo that it ruſhes on and ſtrikes againſt any Obſtacle with that only Difference of Velocity.

AND univerſally, of all Bodies included in a given Space, their Motions amongſt themſelves will be the ſame, their Congreſs the ſame, the ſame Force of their Percuſſion, whether that Space is at reſt, or moves uniformly in a right Line.

HAVING, and that prolixly enough, explained Motion, Reſt, Celerity, as well abſolute as relative, we now come to define other Terms.

VIII. THE *Space paſſed over is the Way that by a Body is run through in its Motion.*

IX. ITS *Length is the right Line that is deſcribed by the Center of the moved Body.*

X. THE *Direction of Motion is the right Line whither the moving Body tends.*

XI. EQUABLE *Motion is when the moving Body deſcribes all the Parts of the Length or Space paſſed over always with the ſame Celerity.*

XII. ACCELERATED *Motion is that, whoſe Velocity increaſes continually.*

XIII. RETARDED *Motion is that, whoſe Velocity is continually diminiſhed.*

XIV. MOTION *equably accelerated is that to which in equal Times there is always added equal Increments of Velocity.*

XV. MOTION *equably retarded is that, whoſe Velocity in equal Times equally decreaſes, even to reſt.*

XVI. A MOMENTUM (*which is often called the Quantity of Motion, and alſo ſimply Motion*) *is that Power or Force incident to moving Bodies, whereby they continually tend from their preſent Places.*

XVII. BUT *an Impediment is that, which obſtructs or reſiſts Motion, and deſtroys, or at leaſt diminiſhes it.*

XVIII. THE *moving Force is the Power of an Agent to cauſe Motion.*

XIX. AN *impreſſed Force is an Action exerted on a Body, to change its State either of Motion or Reſt.*

IF a Body A at reſt is put in motion with a given Celerity, the Force that is impreſſed on it, and by receiving which it begins to move with a given Velocity, is called the impreſſed Force; in which caſe it only differs from the moving Force in the Manner of conceiving it: for the ſame Force, as it proceeds from the Agent, is called the moving Force, and as it is received by the Patient, the impreſſed Force. So likewiſe if a Body B is moving, a certain determinate Force is required to diminiſh its Motion, and alſo a certain determinate Force is neceſſary to put a thorough ſtop to its Motion; which, when it is exerted on the Body B, it is called the impreſſed Force.

I AM not ignorant that there are ſome Philoſophers who do not diſtinguiſh the Quantity of Motion from its Celerity; but ſay thoſe Bodies have equal Motions, that are moved with an equal Celerity, whether the Bodies themſelves are equal or unequal, whether one is very ſmall, and the other ever ſo great, ſo that each Body is carried with the ſame Velocity, they contend that the Quantity of Motion in each will always remain the ſame. But not only Reaſon, but Experience, ſhews that Motion is not only increaſed in the Ratio of the Velocity, but alſo in that of the Bulk or Magnitude, the Bodies being ſuppoſed homogeneal, or of the ſame Species: As, for example, let A and B be two Bodies, A the greater, and B the leſs, and the Momentum or Quantity of Motion of A will be not only greater than the Momentum of B, if A is moved ſwifter than B; but if both are carried with an equal Celerity, the Force or Energy whereby the greater Body A is carried, will be greater than that which the Body B has to change its place; becauſe there is required a greater contrary Force of an Obſtacle or Impediment to ſtop the Motion of the greater Body A, than that which

which is neceſſary to deſtroy the Motion of the leſs Body B. For if the Body A is an hundred Pounds weight, but the Body B one Pound only, and if in both Bodies the Celerities are equal; the Force that the Body A exerts, and whereby it endeavours to remove an Obſtacle, (and therefore the Force of the Impediment reſiſting and deſtroying its Motion, will be much greater than the Force of the Motion of the Body B) *viz.* whereby it endeavours to remove the Impediment, and the Force of that Impediment, which is neceſſarily required to deſtroy the Motion of the Body B, will be leſs than the Force of the Impediment that ſhall be ſufficient to take away the Motion of the moving Body A. But we ſhall hereafter give Theorems, whereby may be eſtimated the Quantity of Motion, and its Meaſure be determined.

XX. THOSE *moving Forces are equal, which acting alike, produce equal Quantities of Motion in a given Time.*

XXI. THOSE *Forces are contrary, whoſe Lines of Direction are contrary.*

XXII. GRAVITY *is a Force bearing downwards, whereby Bodies tend to the Earth in right Lines.*

XXIII. *A* CENTRIPETAL *Force is that Force whereby a Body continually tends to ſome Point as a Center; and hence it follows, that Gravity is a certain centripetal Force.*

XXIV. BUT *by a Centrifugal Force we mean a Force whereby any Body is continually ſollicited to recede from a Center.*

BUT theſe Forces are always eſtimated by the contrary Forces, that are able to keep the Bodies in the ſame ſtate. So if any Body fixed to a String is revolved about an immoveable Center, the Force whereby it endeavours to recede from the Center, is the centrifugal Force; but the Action of the String reſiſting and continually drawing back the Body towards the Center, whereby it comes to paſs that the Body is always retained in the ſame Circle, will be as a centripetal Force equal to the centrifugal one, ſo that one of theſe Forces may be aptly eſtimated by the other. So likewiſe the Force of Gravity of any Body is known by a Force contrary and equal to it, whereby its Deſcent may be hindered. But that Force may be either the weight of another Body acting contrarily by means of a mechanical Inſtrument, as, for example, a Balance, or a centrifugal Force that will ariſe, if that Body revolves in a Circle about the Center of the Earth with ſome certain and determinate Velocity; or laſtly, it may be the Firmneſs and Reſiſtance of another Body on which the preſſing Weight reſts.

XXV. THE *Accelerating Quantity of any Force, is the Meaſure of the Velocity which that Force generates in a given time.*

AT the ſame Diſtance from the Earth, all Bodies, of how unequal weight ſoever they are, do deſcend equally ſwift, and therefore their accelerating Forces are equal; but at unequal Diſtances they are unequally accelerated, *viz.* at a greater, leſs; at a leſs Diſtance, more.

LECTURE VIII.

HAVING finiſhed the Definitions, which were neceſſary to explain ſuch things as are not ſufficiently clear, or thoſe Terms that are leſs uſual, we now come to treat of the Philoſophical Axioms. But ſince the Object of Natural Philoſophy are Bodies and their Actions on one another, which are not ſo eaſily and diſtinctly conceived, as thoſe ſimple Species of Magnitudes which are the Subject of Geometry; I would not have any one, in phyſical Matters, inſiſt ſo much on a rigid Method of Demonſtration, as to expect the Principles of Demonſtrations, that is, Axioms ſo clear and evident in themſelves, as thoſe that are delivered in the Elements of Geometry: for the Nature of the thing will not admit of ſuch. But we think it ſufficient, if we deliver ſuch as we apprehend are congruous to Reaſon and Experience, whoſe Truth ſhines out, as it were, at firſt view, which procure the Belief of ſuch as are not obſtinate, and to which nobody can deny his Aſſent, unleſs he profeſſes himſelf to be altogether a Sceptick.

BUT alſo in demonſtrating, it is neceſſary to make uſe of a more lax ſort of Reaſoning, and to exhibite Propoſitions that are not abſolutely true, but nearly approaching to the Truth. As, for example, when it is demonſtrated that all the Vibrations of the ſame Pendulum made in the ſmall Arches of a Circle, are of equal Duration; it is here ſuppoſed, that the ſmall Arch of a Circle and its Chord are of the ſame Declivity, and of the ſame Length: which however, if we regard the rigid Truth, is not to be admitted;

but in Phyſicks, this Hypotheſis varies ſo little from the Truth, that the Difference ought juſtly to be neglected, and the Diſagreement of the Vibrations ariſing from that difference, is altogether inſenſible, as is proved by Experience. So likewiſe that eminent Philoſopher and Geometer, Dr. *Gregory*, in his *Elements of Catoptricks and Dioptricks*, makes uſe of a more lax Geometry, by aſſuming Lines and Angles as equal, that in reality are unequal, tho' they accede nearly to an Equality: And ſo he ſolves many beautiful Phyſical Problems, which otherwiſe would prove very intricate. And alſo this Method ſeems to be approved of ſometimes by Sir *Iſaac Newton* himſelf, as may be ſeen in *Prop.* 3. *Lib.* 2. of his *Philoſophiæ Naturalis Princip. Math.*

BUT if there are any who harden their Minds againſt ſuch Principles and Demonſtrations, and will not ſuffer themſelves to be convinced by Propoſitions ſufficiently manifeſt; we leave ſuch to enjoy their ſupine Ignorance, nor do we think them worthy to be admitted to the knowledge of the true Philoſophy.

AXIOMS.

I. THERE *are no Properties or Affections of a Nonentity or Nothing.*

II. NO *Body can be naturally annihilated.*

III. EVERY *Mutation induced in a natural Body, proceeds from an external Agent; for every Body is a liſtleſs Heap of Matter, and it cannot induce any Mutation in itſelf.*

IV. EFFECTS *are proportionable to their adequate Cauſes.*

V. THE *Cauſes of Natural Things are ſuch, as are the moſt ſimple, and are ſufficient to explain the Phænomena: for Nature always proceeds in the ſimpleſt and moſt expeditious Method; becauſe by this Manner of operating the Divine Wiſdom diſplays itſelf the more.*

VI. NATURAL *Effects of the ſame kind have the ſame Cauſes: as the Deſcent of a Stone and a Piece of Wood proceeds from the ſame Cauſe; and there is alſo the ſame Cauſe of Light and Heat in the Sun and in a Kitchen-Fire, of the Reflection of Light in the Earth and in the Planets.*

VII. IF *two things are ſo connected together, that they perpetually accompany each other, that is, if one of them is changed or removed, the other likewiſe will be in the ſame manner changed or removed; either one of theſe is the Cauſe of the other, or they both proceed from the ſame common Cauſe.*

So if there be a magnetick Needle moveable on an Axis, upon a Loadſtone being brought near it, and moved round it, the Needle will immediately be moved with the ſame Tenour: and if the Motion of the Loadſtone be ſtopped, the Circulation of the Needle likewiſe ſtops; and it will again begin to revolve, as ſoon as the Loadſtone is again moved: whence nobody can doubt, but that the circular Motion of the Needle depends on the Motion of the Loadſtone. So likewiſe ſince the Flux and Reflux of the Sea happens at the ſame place when the Moon comes to the ſame Hour-Circle, and conſtantly obſerves its Motion; inſomuch that a Period of the Tides ſo preciſely anſwers to the Period of the Moon's Motions, that there has been found no Aberration for ſo many Ages; for it is retarded 48 Minutes every day, and at the Conjunction of the Sun and Moon, the Tides are always at the higheſt, and in their Quadratures at the loweſt: Whence it muſt be granted, that the Flux of the Sea depends on the Motion of the Moon, and its Situation in reſpect to the Sun.

VIII. ANY *Body being moved in any Direction, all its Particles which are relatively at reſt in it, proceed together in the ſame Direction with the ſame Velocity; that is, a relative Place being moved, that which is placed therein will be alſo moved.*

IX. EQUAL *Quantities of Matter carried along with the ſame Velocity, their Momenta or Quantities of Motion will be equal.*

FOR the Momentum of any Body is the Sum of the Momenta of all the Particles compoſing that Body; and therefore where the Magnitudes and Numbers of the Particles are equal, the Momenta will be equal.

X. EQUAL *and contrary Forces acting on the ſame Body, deſtroy their mutual Effects.*

XI. BUT *from unequal and contrary Forces there is produced a Motion Equivalent to the Exceſs of the greater Force.*

XII. *A* MOTION *produced from conſpiring Forces, that is, acting in the ſame Direction, is equivalent to their Sum.*

XIII. IF *what is equivalent be either augmented, or its contrary diminiſhed, then it becomes the greater.*

THEY who would philoſophize mechanically, exhibite the two following.

XIV. ALL *Matter is every where of the ſame Nature, and has the ſame eſſential Attributes, whether it is in the Heavens or on the Earth, whether it appears under the form of a fluid Body, or a hard, or of any other whatever; that is, the Matter of any Body, for example of Wood, does not differ eſſentially from the Matter of any other Body whatever.*

XV. BUT *the different Forms of Bodies are nothing but the different Modifications of the ſame Matter; and depend on the various Magnitude, Figure, Texture, Poſition,*

ſition, and other Modes of the Particles compoſing Bodies.

XVI. So *likewiſe the Qualities, or Actions or Powers of ſome Bodies on other Bodies, ariſe only from the former Affections and Motion conjointly.*

But the Philoſophers ſuppoſe Matter to be the common Subject or *Subſtratum* of all Forms and Qualities; that it is indifferent to them all, ſince it is capable of them all, and remains the ſame under whatever Form it may appear; and whence it is called by the Peripateticks *Materia Prima*, or the firſt Matter.

But although Forms and Qualities are altogether accidental to Matter, yet they neceſſarily and eſſentially belong to Body, which is made up of Form and Matter together: as, for example, although the Matter of Wood is altogether indifferent to this or that Form or Figure and Texture of Particles, which being varied infinite ways, remains the ſame; yet the Wood cannot ſubſiſt without that determinate Modification of Particles, which conſtitutes the Form of a wooden Body; which being removed, the Wood periſhes, and the ſame Matter paſſes into a Body of another kind. But that the Form of a wooden Body conſiſts in the Modification of Particles, is manifeſt upon putting the Wood into the Fire, whereby its Matter is deprived of its Form; for by the Force of the Fire is diſſolved the Connection and Texture of Particles, and ſome part of them paſſes into Smoke and Vapours, and the other is reduced into Aſhes.

Many Examples are alledged by the Philoſophers, to ſhew that the various Magnitudes, Figures, and Textures of the Particles of the ſame Matter may produce various Forms of Bodies, and that various Qualities may ariſe from their various Motion and Poſition; of which we ſhall here give ſome Inſtances.

FIRST of all, when the Particles of Water are rarefied by the Heat of the Sun, they are rais'd out of the Sea aloft into the Air under the form of Vapours; but this new Form is owing to nothing but the changed Situation of the Parts: for by Rarefaction it happens, that the Particles of Water contain in themſelves more, and perhaps larger Spaces, either altogether empty, or only replete with a very rare Æther; whence their Matter takes up a larger Space than an equal Quantity of aerial Matter, and it is rendered leſs intenſively denſe than the Air, and therefore it is forced upwards after the ſame manner as Cork is, when it is immerſed in Water: Nor do theſe Vapours ever ſtop, till they arrive at Air of the ſame Gravity, where they reſt relatively, and compoſe Clouds that are of a thouſand Shapes.

NOW when by the Courſe of the Winds the Air is rendered leſs heavy, the Vapours retaining the ſame Gravity, muſt neceſſarily ſubſide, and in their fall being condenſed by the Reſiſtance of the Air, and driven into a leſs ſpace, they loſe their Form, and falling to the Earth, aſſume the Species of Rain.

MUCH the greateſt part of this is carried to the Sea in Rivers, to be changed again into Vapours; but ſome part of it mixes with the Earth, and being there depoſited, enters into the Roots and Seeds of Trees and Herbs, whence it ariſes into other and new Species of Bodies. And the ſame Rain-Water compoſes different Bodies, as it enters into the different Seeds of things; ſome paſſes into Plantanes, ſome into Graſſes, others into Flowers, others into Oaks, Aſh, Beech, and other innumerable ſorts of Trees and Plants.

NOR in the ſame Plants does the Rain-Water remain altogether ſimilar, ſince all Plants conſiſt of innumerable heterogeneous Parts: ſo in Hemp, for example, the Root is of one form, the Stalk of another,

ther, the ſlender Fibres of another, the Flowers of another, the Seed of another, the Cells containing the Seed of another.

THE Structure of Veſſels in the ſame Hemp is alſo very various; (for, as in an animated Body, every Plant has its Veſſels ſerving for the Circulation of the Humours) and theſe Parts are of very different Properties: the Stalk, for example, is a ligneous Body, and after being dryed, becomes very friable; whilſt the Rind, or the Membrane covering the Stalk, conſiſts of oblong, very ſlender and folding Fibres variouſly connected together.

THE Hemp-dreſſers ſeparate this Membrane from its Stalk, aud after they have handled it a thouſand different ways, they ſpin its Fibres into oblong Threds; and the Poſition and Situation of the Particles being changed, theſe Fibres take another and a very different Form from that which they had in the green Plant.

THEN theſe Threds being wound up, their ſmalleſt Particles continuing the ſame, appear in the ſhapes of Bottoms. The Weavers variouſly connect and weave theſe Threds, and by their Art make out of them Webs, that yield Garments for Men. Theſe, laſtly, being reduced to Linnen Rags, are ſoaked in Water, and by wooden Hammers reduced, as it were, into a ſoft Pulp; which at length, the aqueous Moiſture being dried up, is tranſmuted into Paper, which if it be put into the Fire, is turned partly into a very fine Powder, and partly vaniſhes in Smoke.

BUT all theſe multifarious Forms under which the ſame Matter appears, ariſe from nothing but the changed Figure, Magnitude, and Texture of Particles, and only depend on them.

SO when Metals are melted, the Coherence of their Parts is diſſolved by the Force of the Fire, and the metallick Particles ſeparated from one another, are hurried about with a moſt rapid Motion, whence they aſſume the Form of a fluid Body.

HENCE alſo (as it ſeems) ariſes the Solution of Salts and Metals in Menſtruums: for their Parts are ſeparated from one another by Fermentation, and being reſolved into their leaſt Particles, they are agitated by the Motion of the Fluid itſelf, whence they will appear as fluid Bodies. From theſe Figures, and the other Modifications of Bodies and their Parts, there ariſe many Effects, many Qualities proper to each ſort of Bodies, which muſt neceſſarily be deſtroyed if the Conſtitution of Parts is changed. So of the ſame Matter, as, for example, Iron, may be formed Keys, Plow-ſhares, Files, Saws, and innumerable other Inſtruments accommodated to various Uſes, whoſe Qualities and Effects ſolely depend on their Figures: for whence have Keys the power of opening Doors, but from their Figure, Magnitude, and Congruity of Parts with the Parts of the Locks into which they are put? Whence have Wedges and Plow-ſhares their power of ſplitting and dividing Bodies? Have not the Writers of Mechanicks demonſtrated, that they derive it from their Figure alone? Whence are thoſe ſo regular Motions performed in Clocks or Watches, but from the Wheels being diſpoſed amongſt themſelves, adapted to one another and joined together? Whence, laſtly, is it, that ſuch great Effects are produced by artificial Machines? Certainly the Reaſon is to be deduced only from their Fabrick.

NOR are natural Bodies leſs indebted to the Conſtitution and Modification of their Parts than artificial ones; for all their Operations proceed only from the Motion, Situation, Order, Figure, and Poſition of their Corpuſcles; which being changed in any Body, the Qualities of that Body are alſo actually changed.

If the Superficies of a Body is uneven and rough, it reflects every way the Light incident on it; becauſe the ſuperficial Parts receiving and remitting the Light, are not all in one and the ſame regular Superficies, but are placed in almoſt infinite, and thoſe different Planes: whence the Light falling on theſe various Planes, muſt neceſſarily be reflected every way. Hence Ice, which when it is whole and ſmooth is almoſt of no colour, yet broken into pieces, and having rough and angular Parts, it appears white, *viz.* when it reflects Light copiouſly and on all ſides. The Reaſon is alſo the ſame for Water's growing white, upon its being turned into Froth.

But ſuch is the Structure of moſt viſible Bodies, that their Surfaces ſuffocate part of the Rays that fall upon them, and remit part. If their Surfaces are ſo conſtituted, as to reflect or ſuffocate equally all ſorts of Rays, their Colour would be white or black, or browniſh, betwixt black and white; for the white Colour does no other ways differ from black, than as white Bodies reflect abundance of Rays of all ſorts, but the black only a few. This appears from the Shade of an opake Body, that whilſt the Sun ſhines, is caſt on a white Wall; for that part of the Wall where the Shadow is, as it receives much fewer Rays than the other Parts, it alſo reflects much fewer Rays, ſo that in reſpect to the other Parts it appears black. But if thoſe other Parts of the Wall had not received more Rays than that Part where the Shadow fell, it would be all of the ſame Colour, *viz.* white.

If the Texture of the Superficies is ſuch, that it reflects one ſort of Rays more plentifully, and all the reſt more ſparingly, the Colour of the Superficies will approach to that, which ariſes from the Rays that are reflected in greater number. This may be hence demonſtrated, that the Colour of the ſame

Object will be different, as it receives different ſorts of Rays, the reſt being intercepted; which was firſt diſcovered by the Great Sir *Iſaac Newton.* So if through a Glaſs Priſm red Rays (for ſo we call thoſe that produce a red Colour) are caſt on a blue Object, the Object will change its blue Colour, and put on a red one; but if it receives only yellow Rays, then its Colour will be changed into yellow: if the incident Rays are blue, it will appear blue; and that Colour will be more vivid than all the other Colours, becauſe it reflects many more of theſe Rays, and ſuffocates fewer of them than of the reſt.

If the Superficies of a Body is exactly ſmooth, that is, without any Roughneſs or Unevenneſs; and it reflects from it Rays in great plenty, it will reflect the Rays that proceed from any Object in ſuch a manner as to afford a viſible Image of that Object. And for that reaſon, the Bodies having ſuch Surfaces are called *Speculums.* If the Speculum is a Plane, the Image will be equal to the Object itſelf, and its place will be behind the Speculum at a Diſtance equal to that of the Object itſelf from the Speculum; if the Speculum be a ſpherical Concave, and the Radiant Object is farther diſtant from it than the fourth part of the Diameter of the Sphere, the Image will appear hanging in the Air between the Object and the Speculum, and will be leſs than the Object; if the Object be placed in the Center, the Image will be alſo there, and equal to it; if the Object proceeds beyond the Center towards the Speculum, ſo that its Diſtance from it ſhall be a fourth part of the Diameter of the Sphere, the Image from the Speculum will paſs beyond the Center, and be greater than the Object; but when the Object ſhall have arrived at the Diſtance of one fourth of the Diameter of the Sphere, then the Diſtance of the Image will become infinite; but if it approaches but a very little nearer to the Speculum, the Image will be behind the Speculum, and greater than the Object itſelf. All theſe Phenomena that are ſo very different, proceed only from the Alteration of Diſtances, every thing beſides remaining the ſame.

LET us now conſider thoſe various and altogether contrary Effects, which ariſe from an Alteration only in Situation and Poſition, every thing elſe being in the ſame State, except ſuch things as depend on the Mutation of the Situation.

ALL Philoſophers now acknowledge the Sun to be at reſt in the Center of his Syſtem, but the Earth, like the other Planets, to be carried about him in the ſpace of a Year; yet the Earth moves about the Sun in ſuch a manner, that its Axis remains always parallel to itſelf, not indeed perpendicular to the Plane of the Earth's Orbit, but inclined to it in an Angle of 66°. 30′. And by reaſon of this Paralleliſm and Inclination, it follows that the Earth muſt ſometimes turn one of his Poles towards the Sun, ſometimes the other; and therefore all the Parts of the Earth will have different Situations, in reſpect of the Sun. From this Change in its Situation, ariſe all thoſe Viciſſitudes of Seaſon that yearly happen, *viz.* Summer, Winter, Spring, and Autumn: for if the Axis of the Earth were perpendicular to the Plane of its Orbit, there would be no Alterations of Seaſons, no Differences in the Days and Nights, but every Part of the Earth would always receive, after the ſame manner, the equal Forces of the Sun's Rays.

BUT ſince every part of the Earth changes its Situation in reſpect of the Sun, and receives its Rays now more, at another time leſs oblique, now for a ſhorter duration, then for a longer; there will thence ariſe different and altogether contrary Appearances. As in the Autumn the Corn becomes ripe, and the Fruits mellow; yet the Fields by degrees loſe their green and pleaſant Colour, and the Leaves fall from the Trees. Afterwards the Winter approaching, all things become cold and horrid, the Snow covers the higheſt Mountains, and the Trees labour with its weight; and what is wonderful, the Waters of the Sea become ſtable and firm, and what before was only paſſable by Ships, now bears Tents and Armies.

But the Earth continually revolving in its Orb, each Part thereof changes its Situation in reſpect of the Sun, and what before was turned from, begins now to reſpect the Sun: and in the mean while the Snows diſappear, the Fields are clothed with Graſs, and the Trees adorned with Leaves, the Horſe now no longer abides in the Stable, nor the Plowman at the Fire-ſide; but a new and joyful Face of things appears, and the Year returns through the Summer to the Autumn.

Now ſince ſo many different, ſo many contrary Effects happen from the ſole Alteration of the Situation, and ſo various Phenomena follow from it, all the other Cauſes remaining the ſame; certainly from the Poſition, Diſtance, Magnitude, Figure and Structure of the Parts compoſing Bodies, from the Motion and Subtility of Effluvia, from the Congruity of Bodies, and their Reſpect to other Bodies; I ſay, from all theſe joined and combined together variouſly, and almoſt after infinite manners, may proceed almoſt infinite different Forms, Affections, and Operations amongſt themſelves of Bodies, nor is any thing to be found in Nature, that does not depend on theſe. For if theſe are changed, the Forms, Qualities, and Operations of Bodies will be alſo changed. For Example, it is certain that the magnetick Attractions and Directions ariſe from the Structure of Parts; for if a Loadſtone be ſtruck hard enough, ſo that the Poſition of its internal

Parts be changed, the Pole of that Loadſtone will alſo be thereby changed. And if a Loadſtone be put into the Fire, inſomuch that the internal Structure of its Parts be changed or wholly deſtroyed, then it will loſe all its former Virtue, and will ſcarce differ from other Stones.

But although it is in the general ſhewn, that the magnetick Operations do in ſome ſort proceed from the internal Conſtitution of Parts, yet the Modus of Operation, deduced from mechanical Principles, and ſuch as are eaſy to be underſtood, is not yet diſcovered. And what ſome generally boaſt of, concerning Effluvia, a ſubtile Matter, Particles adapted to the Pores of the Loadſtone, *&c.* does not in the leaſt lead us to a clear and diſtinct Explication of theſe Operations: but notwithſtanding all theſe things, the magnetick Virtues muſt be ſtill reckoned amongſt the occult Qualities.

From what has been ſaid, it follows, that the Qualities of Bodies which do not depend on their Forms, and which, the Quantity of Matter remaining the ſame, are not increaſed or remitted, but are in all ſorts of Bodies upon which we can make any Experiments; are the univerſal Qualities of all Bodies. For ſince they do not proceed from the Form or Modifications of Bodies, they muſt depend on the Matter itſelf; but ſince the Nature of all Matter is the ſame, and any part of it differs from another only by ſome Modes, the Qualities not produced by theſe Modes will be the ſame in all Matter.

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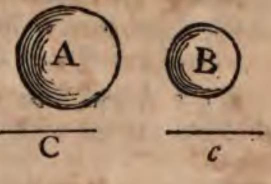
LECTURE IX.

Theorems of the Quantity of Motion, and of Spaces paſſed over by Bodies in Motion.

THEOR. I.

IN *comparing the Motions of Bodies, if the Quantities of Matter of Bodies in motion are equal, the Momenta or Quantities of Motion will be as the Velocities.*

LET A and B be two Bodies in motion, having equal Quantities of Matter, and let A be moved with the Celerity C, but B with the Celerity *c*; I ſay, the Momentum or Quantity of Motion in the moveable Body A, is to the Momentum or Quantity of Motion in the moveable Body B, as the Celerity C to the Celerity *c*: for if any Force is impreſſed on the Body A to move it with a given Velocity C, there is required a double Force to move the Body B with a double Velocity, and a triple Force muſt be uſed to move it with a triple Velocity, and only half that Force is neceſſary to move B with half the Velocity, and ſo of other Multiples and Submultiples. That is, ſince [by the fourth Axiom] Effects are proportionable to their adequate Cauſes, if the Force that is uſed to move



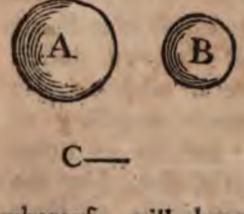
the Body B, is double that which is applied to move A, the Momentum of that will be alſo double of the Momentum of this; if the Force is triple, the Motion of the Body B will be likewiſe triple the Motion of the Body A; if but half the Force is impreſſed on the Body B, its Momentum will be half the Momentum of A: that is, ſince the Velocity of the Body A is univerſally to the Velocity of B, as the Force impreſſed on the Body A to the Force impreſſed on B; and as the Force impreſſed on the moveable Body A to the Force impreſſed on the Body B, ſo is the Momentum or Quantity of Motion in A, to the Momentum or Quantity of Motion in B, the Velocity of the Body in motion A will be to the Velocity of the Body in motion B, as the Motion of A to the Motion of the moveable Body B. *Q. E. D.*

Cor. If the Momenta are as the Velocities, the Quantities of Matter in the moveable Bodies will be equal.

THEOR. II.

In *comparing Motions, if the Celerities are equal, the Momenta or Quantities of Motion of Bodies will be as the Quantities of Matter in thoſe Bodies; or if the moveable Bodies are homogeneal, as their Magnitudes.*

Let A and B be two Bodies in motion, and each of them carried with the ſame Celerity C; I ſay the Momentum of the Body A, is to the Momentum of the Body B, as the Quantity of Matter in A to the Quantity of Matter in B. For if the Quantity of Matter in A is double that in B, A may be divided into two parts, each whereof, will have as much Matter, and conſequently [by Axiom 9.] as much Motion as B has: *viz.*



viz. ſince each Body is moved with the ſame Velocity, ſo that the Momentum of the Body A will be double the Momentum of the Body B. If the Quantity of Matter in A is triple of that in B, A may be divided into three parts, each whereof will have a Quantity of Motion equal to that which is in B; and univerſally whatever proportion the Matter in A has to the Matter in B, the ſame proportion will have the Momentum of A to the Momentum of B, if each Body is carried along with the ſame Velocity.

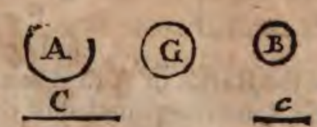
IF the Bodies are homogeneal, their Quantities of Matter will be as their Magnitudes or Bulks; and conſequently their Motion will be alſo in the ſame Ratio of their Magnitudes.

Cor. IF the Momenta are as the Quantities of Matter, the Celerities of the Bodies will be equal.

THEOR. III.

IN *comparing the Motions of any Bodies, the Ratio of the Momenta is compoſed of the Ratios of the Quantities of Matter, and of the Celerities.*

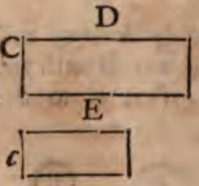
LET A and B be any two moveable Bodies, and let A be moved with the Celerity C, but B with the Celerity *c*; I ſay, the Momentum of A is to the Momentum of B, in a Ratio compounded of the Ratio of the Quantity of Matter in A, to the Quantity of Matter in B, and of the Ratio of the Celerity of the Body A, to the Celerity of the Body B. Let there be a third Body G, whoſe Matter is equal to that which is in A, but let it be moved with the Celerity of the Body B. From the Elements the Ratio of the Momentum of the Body A, to the Momentum of the Body B, is compounded of the Ratio of the Momentum of the Body A to the Mo-



Momentum of the Body G, and of the Ratio of the Momentum of the Body G to the Momentum of the Body B: but [by Theor. 1.] the Momentum of the Body A is to the Momentum of the Body G, as the Celerity C is to the Celerity *c*; and ſince G and B are moved with the ſame Celerity, the Momentum of the Body G will be to the Momentum of the Body B, as the Quantity of Matter in G or A to the Quantity of Matter in B. And therefore the Momentum of the Body A will be to the Momentum of the Body B, in a Ratio compounded of the Celerity C to the Celerity *c*, and of the Quantity of Matter in A or G, to the Quantity of Matter in B. *Q. E. D.*

Cor. 1. If the Bodies are homogeneal, the Ratio of the Momenta will be compounded of the Ratio of the Magnitudes and of the Celerities.

Cor. 2. If it be made as A to B, that is, as the Quantity of Matter in A to the Quantity of Matter in B, ſo is the right Line D to the right Line E; and let be compleated the Rectangles under D and C, and under E and *c*, the Momentum of the moveable Body A will be to the Momentum of the moveable Body B, as the Rectangle D C to the Rectangle E *c*.



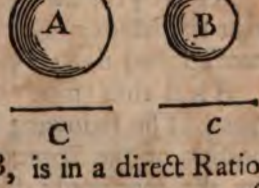
For becauſe it is as A to B, ſo D to E, the Ratio compounded of the Ratios of A to B and of C to *c*, will be equal to the Ratio compounded of the Ratios D to E and of C to *c*; but [by 23 *El.* 6.] the Ratio compounded of the Ratios of D to E and of C to *c*, is equal to the Ratio of the Rectangle DC to the Rectangle E *c*, and [by this third Theor.] the Ratio of the Momentum of the moveable Body A to the Momentum of the moveable Body B, is equal to the Ratio compounded of the Ratios of A to B, or of D to E, and of C to *c*; wherefore it will be as the Rectangle D C to the

Rectangle E *c*, ſo is the Momentum of the moveable Body A to the Momentum of the moveable Body B. The Momentum therefore of any Body may be conſidered as a Rectangle produced from the Multiplication of the Bulk or Quantity of Matter contained in that Body into its Celerity.

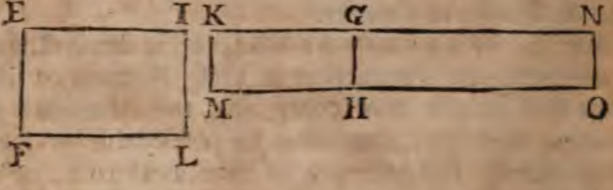
Cor. 3. WHEREFORE whatever is demonſtrated concerning the Proportion of theſe Rectangles, the ſame will be alſo true concerning the Momenta of Bodies, their Momenta being proportionable to theſe Rectangles. For example, if it be as D to E, or as A to B, ſo is *c* to C, in this caſe the Momenta of the moveable Bodies will be equal: for rectangular Parallelograms having their Sides reciprocally proportionable, are [by 14 *El.* 6.] equal; and on the contrary, if the Rectangles are equal, their Sides will be reciprocally proportionable: that is, if the Quantities of Matter, or in Bodies of the ſame kind, their Magnitudes are reciprocally proportionable to their Celerities, their Momenta will be equal; and converſly, if the Momenta are equal, it will be as the Quantity of Matter in one to the Quantity of Matter in the other, ſo reciprocally the Celerity of this to the Celerity of that: hence alſo may be demonſtrated the following

THEOR. IV.

IN *compared Motions, the Ratio of the Celerities is compounded of the direct Ratio of the Moment, and of the reciprocal Ratio of the Quantities of Matter.*



LET A and B be two moveable Bodies, and let A be carried along with the Celerity C, but B with the Celerity *c*: I ſay, C is to *c*, that is, the Celerity of one of the Bodies A to the Celerity of the other B, is in a direct Ratio of the Momentum of the Body A to the Momentum of the Body B, and a reciprocal Ratio of the Matter in A to the Matter in B, that is, directly as the Matter in B to the Matter in A. Let it be as A to B, ſo the right Line E I to the right Line K G; and let I L be equal to C, but G H equal to *c*; and let be compleated the Rectangles E L, K H. By what has been ſaid above, the Rectangles E L, K H, will repreſent the Momenta of the moving Bodies A and B reſpectively: to G H let be applied the Rectangle H N equal to the Rectangle E L. Since therefore H N is equal to E L, it will be [by 16 *El.* 6.] as I L to G H, ſo is G N to E I; but the Ratio of G N to E I is equal to the Ratio of G N to G K, and of G K to E I; that is, equal to the Ratios of the Rectangle H N or E L to the Rectangle K H, and of G K to E I. Wherefore the Celerity C or I L will be to the Celerity *c* or G H, in a Ratio compounded of the Ratio of the Momentum E L to the Momentum K H, and of the Matter G K to the Matter E I: that is, the Velocity of any Body is always as its Momentum applied to its Matter. *Q. E. D.*



AND by the like manner of Reaſoning it is found, that the Matter of any Body is always as the Momentum applied to its Velocity.

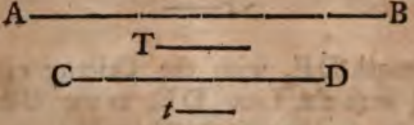
AND thus much concerning the Momenta of Bodies. The following Theorems concerning the Proportion of the Spaces paſſed over by Bodies in motion, are alſo commonly demonſtrated.

THEOR.

THEOR. V.

IN *compared Motions, if the Celerities of the moving Bodies are equal, the Spaces run over by them, will be directly as the Times wherein the Motions are performed.*

LET a Body in motion run the length A B, in the Time T, with an equable and uniform Motion; alſo



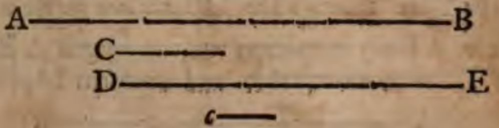
let the ſame or another Body in motion, carried along with the ſame Velocity, run the length C D, in the Time: I ſay, the Line A B is to the Line C D, as the Time T to the Time *t*. For if the Time T is double that of *t*, it may be divided into two Parts, each whereof will be equal to *t*; ſo that each Space run over with the ſame Velocity in theſe equal Parts of Time, will be equal to the Space run over in the Time *t*, and the two Spaces taken together will be double the Space run over in the Time *t*. In the ſame manner, if T is triple of *t*, T may be divided into three equal Parts, and the Spaces run over in each of theſe Times, will be equal to the Space run over in the Time *t*; and therefore the three Spaces taken together, will be triple the Space run over in the Time *t*. The ſame may be ſhewn of other Multiples and Submultiples: wherefore univerſally, whatever proportion T has to *t*, the ſame will the Space run over A B have to the Space run over C D. *Q. E. D.*

Cor. IF the Times are as the Spaces run over, the Celerities will be equal.

THEOR. VI.

IN *compared Motions, if the Times of the Motions are equal, the Spaces run over will be as the Celerities.*

Let any moving Body run in a given Time the length A B, with the Celerity C; and in the ſame, or an equal Time, let the ſame or another moving Body



run the length D E, with the Celerity *c*; I ſay the Line A B is to the Line D E, as the Celerity C to the Celerity *c*. For if the Celerity C is double of *c*, the Space A B run over with the Celerity C, will be double the Space D E run over with the Celerity *c*; if the Celerity C is triple of *c*, the Length A B will be alſo triple the Length D E; if C is the half of *c*, A B will be the half of D E: and univerſally, ſince equal Times are ſpent in running over the Lines, whatever proportion the Celerity C has to the Celerity *c*, the ſame will the Length run over A B have to the Length run over D E. *Q. E. D.*

Cor. If the Celerities are as the Spaces run over, the Times will be equal.

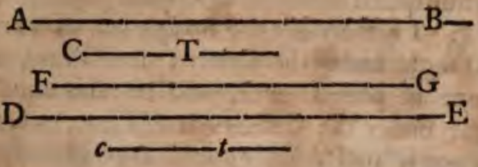
The two firſt Theorems, as alſo the fifth and this ſixth, might be demonſtrated univerſally by Equimultiples, after the manner of *Euclid*; but ſince they are ſo evident, that they may be looked upon as Axioms, they ſcarce need ſo great an Apparatus of Demonſtration.

THEOR. VII.

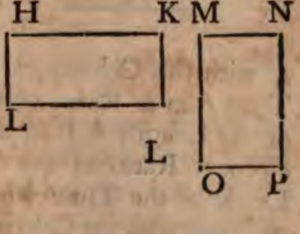
The *Lengths run over are in a Ratio compounded of the Ratios of the Times and of the Celerities.*

Let the Line A B be run over with the Celerity C, in the Time T; and the Line D E with the Celerity *c*, in the Time *t*: I ſay, the Ratio of A B to D E

D E is compounded of the Ratio of the Celerity C to the Celerity *c*, and the Ratio of the Time T to the Time *t*. Let the Line F G be run over in the Time T, with the Celerity *c*; it is manifeſt, that A B is to D E in a Ratio compounded of the Ratios of A B to F G and of F G to D E. But becauſe A B and F G are run over in the ſame time, it will be as A B to F G, ſo the Celerity C to the Celerity *c*; but ſince the moving Bodies deſcribe the Lines F G and D E with the ſame Celerity, it will be [by Theor. 6.] F G to D E as the Time T to the Time *t*. Wherefore ſince the Ratio of A B to D E is compounded of the Ratios of A B to F G, and of F G to D E, it will likewiſe be compounded of the Ratios that are equal to theſe Ratios, *viz.* of the Ratio of the Celerity C to the Celerity *c*, and of the Time T to the Time *t*.



Cor. 1. If H K be made equal to C, H I equal to T, alſo M N equal to *c*, and M O equal to *t*, and let be compleated the rectangular Parallelograms H L, M P; it will be A B to D E, as the Rectangle H L to the Rectangle M P: for [by 23 *El.* 6.] the Rectangle H L is to the Rectangle M P, in a Ratio compounded of the Ratios of H K to M N, and of H I to M O; but [by the preceding Theorem] the Space run over A B is



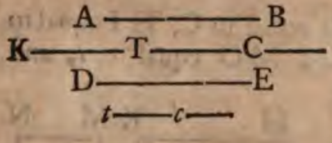
is to the Space run over D E, in a Ratio compounded of theſe Ratios: whence theſe Spaces run over may be conſidered as Rectangles produced from the Times multiplied into the Celerities.

Cor. 2. If therefore the Spaces run over are equal, the Rectangle under the Celerity and Time, wherein one Space is paſſed over, will be alſo equal to the Rectangle under the Celerity and Time, wherein the other Space is paſſed over; and conſequently it will be as the Celerity to the Celerity, ſo reciprocally is the Time to the Time, [by 14 *El.* 6.] that is, if the Spaces run over are equal, the Times will be reciprocally as the Celerities.

THEOR. VIII.

In *compared Motions, the Ratio of the Times is compounded of the direct Ratio of the Length, and of the reciprocal one of the Celerities.*

This Theorem may be demonſtrated from the former, after the ſame manner as the fourth follows the third: but for the ſake of Perſpicuity, we ſhall thus briefly evince it. Let the Length A B be run over in the Time T, with the Celerity C; alſo let the Length D E be run over in the Time *t*, with the Celerity *c*; I ſay, the Time T is to the Time *t* in a Ratio compounded of the direct Ratio of the Length A B to the Length D E, and of the reciprocal Ratio of the Celerity C to the Celerity *c*. Let K be the Time wherein the Length A B may be run over with the Celerity *c*, the Ratio of the Time T to the Time *t*, will be compounded of the Ratio of T to K, and of K to *t*; but [by the Corol. of the preceding Prop.] it is as T to K, ſo *c* to C (ſince the



the ſame Space is run over in each of the Times) and as K to t, ſo is (by the Cor. of Theor. 5.) the Length AB to the Length DE. Wherefore T will be to t in a Ratio compounded of the Celerity c, to the Celerity C, and of the Length AB to the Length DE; that is, the Times are in a Ratio compounded of the reciprocal Ratio of the Celerities, and the direct one of the Lengths. *Q. E. D.*

AFTER the ſame manner may be ſhewn, that the Celerities are in a direct Ratio of the Lengths, and a reciprocal one of the Times.

Cor. 1. AND hence it follows, that the Time is as the Space run over applied to the Celerity.

Cor. 2. The Celerity likewiſe is as the Space run over applied to the Time.

THE third and ſeventh Theorem may be demonſtrated from this univerſal Theorem, *viz.*

IF any Effects depend at the ſame time on ſeveral Cauſes, in ſuch a manner, that as they are augmented or diminiſhed in the ſame Ratio wherein any of the Cauſes are augmented or diminiſhed, thoſe Effects will be in a Ratio compounded of all the Cauſes: that is, if the Cauſes A, B, C, acting together do produce the Effect E, which, all the reſt remaining the ſame, is as ſome of the Cauſes; and if other Cauſes a, b, c, reſpectively like the former, and acting after the ſame manner, do produce the Effect e; it will be as E to e, ſo $A \times B \times C$ to $a \times b \times c$. Which may be eaſily ſhewn, by almoſt the ſame Method that we uſed in the preceding Demonſtrations.

AFTER the like manner, if the ſame Effect depends on ſeveral things at once, whereof ſome aſſiſt or augment the ſame, in that Ratio whereby they are augmented; but others impede or diminiſh it, in the ſame Ratio whereby they are augmented: the Effect will always be directly as the aſſiſting Cauſes, and reciprocally as the impeding or diminiſhing Agents.

The ſeventh Theorem is thus demonſtrated in the Style of Sir *Iſaac Newton*:

The *Celerity being given, the Space run over is as the Time; and the Time being given, the Space run over is as the Celerity: wherefore neither of them being given, it is as the Celerity and Time conjointly.*

So likewiſe may the eighth Theorem be ſhewn.

The *Celerity being given, the Time is directly as the Space run over; and the Space being given, the Time is reciprocally as the Celerity: wherefore neither of them being given, the Time will be directly as the Space and reciprocally as the Celerity.*

In like manner may the third and fourth Theorem be explained, and we ſhall ſometimes make uſe of this Method for the ſake of Brevity.

LECTURE X.

In the Demonſtrations of the preceding Lecture we have ſhewn a Method, whereby Phyſical Matters may be firſt reduced to Geometry, and afterwards to Arithmetick: for ſince it is there demonſtrated, that the Motions of Bodies are as Rectangles under their Celerity and Quantity of Matter; from the Celerity and Quantity of Matter of any Body being given, there will be given its Momentum, *viz.* equal to a *Factum* or Product made of the Celerity of the Body into its Quantity of Matter. As, for example, let the Body A be 8 parts, but B be 6 parts, the Celerity of A as 5, and the Celerity of the Body B as 3; the Motion of the Body A will be 40, and the Motion of the Body B but 18 Parts.

So from the given Momentum and Quantity of Matter of any Body, may be known its Celerity; *viz.* if its Momentum be divided by its Quantity of Matter, the Quotient will give its Velocity. For let the Motion in the Body A be 40 parts, and its Quantity of Matter 8 parts; let alſo the Motion in the Body B be 18 parts, and its Quantity of Matter 6 parts: divide 40 by 8, the Quotient will give 5, *viz.* the Velocity of the moving Body A; and by dividing 18 by 6, the Quotient will give 3, the Velocity of the moving Body B.

Since by Examples things become more plain, and Numbers are to be made uſe of in Practice, that the young Beginners may accuſtom themſelves the better to theſe, let us illuſtrate the Science of Motion by Numbers, and make uſe of both ſpecious and numerical Arithmetick; for from ſpecious Arithmetick are diſcovered general Canons, which afterwards may be applied to particular Numbers.

So let A denote the Quantity of Matter in any given Body B, but C ſtand for its Celerity, and let its Momentum be called M; or rather let theſe Letters be put for the Numbers that are proportionable to thoſe Quantities: then it will be $C \times A = M$ and $C = \frac{M}{A}$ and $A = \frac{M}{C}$.

In like manner, ſince the Space run over is always proportionable to the Rectangle under the Celerity and the Time; if the Space be called S, the Time T, and Celerity C, it will be $S = C \times T$; and $C = \frac{S}{T}$; and $T = \frac{S}{C}$; and therefore ſince it is $M = A \times C$, it will alſo be $M = \frac{A \times S}{T}$: or if T is given, it will be $M = A \times S$; that is, the Momentum of any Body is as its Matter drawn into the Space run over by it in a given Time. Many other Concluſions likewiſe, which ſome deliver as the Laws of Motion, may be deduced from what has hitherto been demonſtrated; but ſince any Tyro may eaſily of himſelf diſcover them all, it is not worth while to produce them here.

FROM what has been demonſtrated, it is manifeſt, that the Momentum of any Body ariſes from the Motion of all its Parts: for in each Particle of the Body there is an Impetus or Force of moving, and from the Sum of theſe Forces is compounded the Impetus or Quantity of Motion of the whole Body.

HENCE alſo may be gathered, that by how much the greater is the Quantity of Matter in Bodies, by ſo much the greater ought the Force to be, in order to move thoſe Bodies with a given Velocity, and therefore their Momenta will be greater in the ſame Ratio. If therefore there are two Bodies carried along with the ſame Velocity, the Quantities of Matter in them will be always as their Momenta; ſo that if Bodies which are equal in Bulk, and moved with the ſame Velocity, have unequal Momenta, it is neceſſary that the Quantities of Matter in them are unequal: and that which has the leſs Momentum, will have more Pores or Spaces either altogether empty, or replete with ſome Matter which does not partake of the Motion of the whole Body, whoſe Pores it is ſuppoſed to fill. So, for example, if there are two Globes, one of Cork, the other of Lead, both of the ſame Magnitude, and moved with the ſame Velocity, ſince it is known by Experience, that the Momentum of one is much greater than the Momentum of the other, it muſt follow that there are many more Pores in one than in the other; which it muſt be allowed, are either altogether empty, or replete with ſome moſt ſubtile Matter, which may ſo freely paſs through its Pores, as not to partake of the Motion of the Body whoſe Pores it poſſeſſes.

BUT that this Matter may pervade the Pores of other Bodies, without partaking of their Motion, it is neceſſary that all Bodies ſhould have all their Pores extended in right Lines parallel to the Direction of Motion; *viz.* that there may be no Reflections of the ſubtile Matter againſt the Sides of the Pores; otherwiſe the Matter, although ever ſo ſubtile, will be moved together with the Body, whoſe Pores it is ſuppoſed to fill. The ſubtile Matter therefore cannot but partake of the Motion of the Body, unleſs the Body moved be ſo diſpoſed, that it has its Pores parallel to the Direction of Motion; but ſince its Situation may be varied infinite other ways, that is, the Lengths of the Pores may be inclined to the Line of Direction in infinite Angles; and therefore all theſe things being ſuppoſed, whilſt the Body is moving, the ſubtile Matter placed in its Pores will be moved along with it: therefore a ſubtile Matter cannot ſo freely pervade the Pores of Bodies, but it muſt partake of their Motion; conſequently the Body being moved, the Matter alſo contained in it will be moved, however ſubtile it be. If therefore a Body of Cork is moved, it carries along with it the Matter contained in its Pores; ſo that ſince it has a leſs Momentum than a Globe of Lead of the ſame Magnitude, and moved with the ſame Velocity, the Quantity of Matter in the Body of Cork will be leſs, and conſequently there will be more Pores or Spaces abſolutely empty.

FROM what has been demonſtrated, may be deduced the following Theorem.

THEOR. IX.

THE *Weights of all ſenſible Bodies near the Superficies of the Earth, are proportionable to the Quantities of Matter contained in them.*

FOR, as is manifeſt from many Experiments of Pendulums, all Bodies falling perpendicularly by the Force of Gravity (abſtracting the Reſiſtance of the Air) deſcribe equal Spaces in the ſame Time. For *in vacuo*, or a Medium that has no Reſiſtance, the ſmalleſt Feather takes up no more time in falling than a heavy Piece of Lead; ſo that the Velocities of all Bodies falling in a given Time, are equal; their Momenta therefore will be proportionable to the Quantities of Matter contained in them: but the Forces generating Motion are always proportionable to the Motions or Momenta generated, and therefore in this caſe they will be as the Quantities of Matter in the Bodies moved; but the Forces that generate theſe Motions, are the Gravitations of Bodies, that is, their Weights. Therefore the Weights of all Bodies are proportionable to the Quantities of Matter contained in thoſe Bodies. *Q. E. D.*

Cor. 1. THE Weight therefore of any Body, will be augmented or diminiſhed, from the Quantity of its Matter being only augmented or diminiſhed.

Cor. 2. WHEREFORE the Quantity of Matter continuing the ſame in any given Body, its Weight will alſo continue the ſame; and however its Figure, or the Texture of the Particles compoſing that Body be varied, yet its Weight will not be changed: ſo that the Weight of no Body depends on its Form or Texture.

SINCE [by Axiom 14.] the Nature of all Matter is the ſame, neither does one Body differ from another, but in its Modes, as by the Figure of its

Parts, Situation, and the like Forms; the Affections of Bodies, which do not depend on their Forms, will be the ſame in all Bodies: ſo that ſince (as has been ſaid) the Weights of Bodies do not ariſe from their Forms, but depend on their Quantity of Matter; in equal Quantities of Matter, at the ſame diſtance from the Earth, the Gravitations towards the Earth will be equal: but if the Weights of two Bodies are unequal, the Quantities of Matter contained in them will be alſo unequal.

LET us ſuppoſe now two Globes of equal Magnitudes, the one of Lead, the other of Cork; if the Quantity of Matter in both was the ſame, (by what has been ſhewn) both Bodies would equally ponderate: for the ſubtileſt Matter occupying the Pores of the Cork, would ponderate equally with the Matter of Lead that is equal to it. But ſince there is a great difference in the Weights of theſe two Bodies, there will be alſo a great difference in the Quantity of their Matter; and if Lead is thrice heavier than Cork, the Matter contained in the Lead will be triple of that in the Cork: ſo that there will be more Pores or Spaces abſolutely empty in the Cork, than in the Lead. A *Vacuum* therefore is not only poſſible, but actually given; which was to be proved. And hence it follows, that the Quantity of Matter in any Body may be properly eſtimated by its Gravity.

SINCE the Momentum may be augmented, as well by increaſing the Quantity of Matter, the Velocity remaining the ſame, as by increaſing the Velocity, the Quantity of Matter remaining the ſame; the Antients (who were ignorant of the Force of Gunpowder to give a ſwift Motion to Bodies) to beat down their Enemies Walls, made uſe of Machines ſo framed, that a huge Maſs of Matter might batter the Walls, though not with a great Velocity, yet with a violent Impetus: but now-a-days ſmall Bullets are hurled out of Cannon with a great Velocity, by the Exploſion of Gunpowder. But although the warlike Engines of the Antients were much inferiour to ours, yet their Force to overturn Walls was almoſt incredible: for their Battering-Rams were compoſed of vaſt Beams joined together, whoſe Weight may be gathered from hence, that ſome of them required ſix thouſand Men (*viz.* that ſome might ſucceed others) to direct them, and to give them their Motion. That part, whereby the Wall was battered, was made of ſolid-Iron, and they were ſo flung by Ropes, (I mean the compounded Rams) that their Lengths were parallel to the Horizon; whence being drawn backwards by a great number of Men, and immediately driven forwards by their Weight, and the Forces of the Men acting together, they ſtruck the Walls with their Iron End: and, according to the Teſtimony of *Joſephus*, there were no Towers ſo ſtrong, or Walls ſo thick, as to ſuſtain their conſtant Shocks.

IN Machines that raiſe Weights by the Circumgyrations of Wheels, ſometimes the Wheels are rendered heavier by the addition of Lead, *viz.* that the greater Quantity of Matter may receive the greater Impetus or Quantity of Motion; whereby it may the better oppoſe the Reſiſtance, ariſing as well from the Air as from the Friction of the Matter, and the longer preſerve their Motion; which therefore once begun, will be eaſily continued.

FROM the ſame Principle alſo it is, that Spinners by putting on heavy Wharls on their Spindles, their Gyrations are continued longer: ſince the Part of the Motion loſt by the Reſiſtance of the Air, bears a leſs proportion to the Motion increaſed by the addition of Matter, than that which it had to the Motion not thus increaſed.

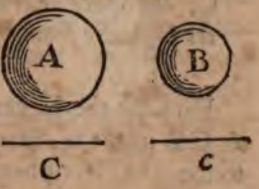
FROM what has been ſaid may be ſolved the following Problem.

PROBL.

PROBL. I.

To find the Velocity, wherewith a given Body muſt be moved, ſo that it may have a Momentum equal to any given Momentum.

LET the given Body be A, whoſe Momentum ought to be equal to the Momentum of the Body B, moved with the Celerity *c*; let it be as A to B, ſo the Celerity *c* to another Quantity C; this will be the Velocity ſought, wherewith if the Body A is moved, its Momentum will be equal to the Momentum of the Body B, as is manifeſt from the third Corollary of the third Theorem. For the Momenta of Bodies are equal, if their Celerities are reciprocally proportionable to thoſe Bodies; but, by Hypotheſis, the Celerity of the Body B is to the Celerity of the Body A, as the Body A to the Body B: whence the Momentum of the Body A will be equal to the Momentum of the Body B. *Q. E. I.*



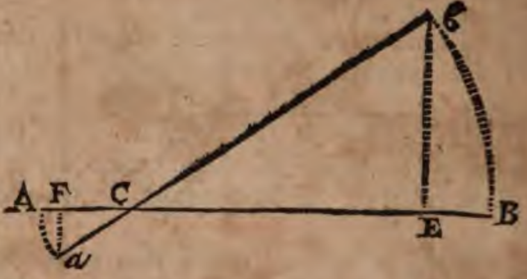
AND hence it follows, that a Body, tho' ever ſo ſmall, may have a Momentum equal to the Momentum of a Body, tho' ever ſo great, that is moved with a given Velocity. On this Principle depend all the Engines which are contrived to draw or raiſe Bodies: for if the Engines are ſo diſpoſed, that the Velocity of the Power ſhall be to the Velocity of the Weight, as that Weight is to the Power; I ſay, in that caſe the Power will ſuſtain the Weight. We will illuſtrate this in the five ſimple mechanick Machines. And firſt of all in the Lever, which we ſhall here conſider as an inflexible Line, whether a right one or a curve, or compoſed of many right Lines, that is moveable about a fixed Point, of no Gravity indeed of itſelf, yet accommodated to ſuſtain and raiſe Weights.

The fixed Point by which the Lever is ſuſtained, and on which it moves round, is called its *Fulcrum* or Prop.

THEOR. X.

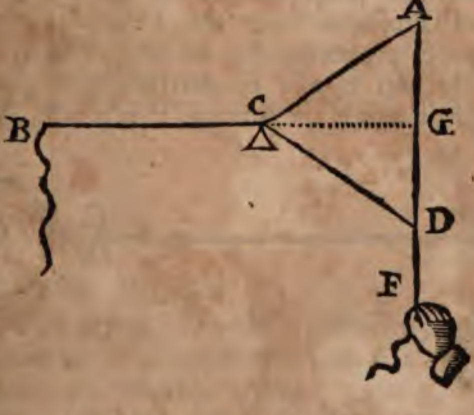
Let AB *be a Lever only moveable round the Fulcrum* C, *the Space deſcribed by each of its Points, will be as its Diſtance from the Fulcrum.*

For let the Lever be moved out of the Situation A CB into the Situation α Cβ, the Point A will deſcribe the Periphery A α, but B will paſs over the Periphery Bβ. Now by reaſon of the ſimilar Sectors A Cα, B Cβ, A α is to B β, as A C to B C; that is, the Spaces deſcribed by the Points A and B, are as their Diſtances from the Fulcrum. If to the Points A and B be applied Powers drawing the *Brachia* of the Lever perpendicularly; the Spaces that are deſcribed by them according or contrary to their Propenſions, are not the Peripheries A α, B β, but the Perpendiculars α F, β E, let fall on the Brachia of the Lever. For the Power in A is moved, according to its proper Direction or Propenſion, through the Space α F only, and no farther: as, for the ſame cauſe



cauſe, the way paſſed through by the Power B, according to its proper Direction, is to be eſtimated by β E. But by reaſon of the equi-angular Triangles α C F, β C E, α F is to β E as α C or A C to β C or B C; that is, the Spaces run over by Powers according to their proper Directions, will be as their Diſtances from the Fulcrum.

But if the Direction of the Power is not a right Line, perpendicular to the Brachium of the Lever A C, let from the Fulcrum to the Line of Direction be drawn the Perpendicular C G, and the Space deſcribed by the Power according to its Propenſion, will be proportionable to that Perpendicular: for it matters not, whether the Thred F G A, by which



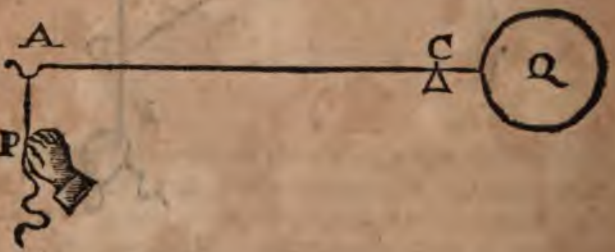
the Power acts, is affixed to the Point G or A, or indeed to the Point D; for the Line of Direction remaining the ſame, its Force to move round the Plane A D C B will be the ſame, as if the Thred was fixed to the Point G, and the Way deſcribed by it in a given Time, according to its proper Direction, will be proportionable to the right Line C G. Wherefore it is manifeſt in every caſe, that the Way de-

deſcribed by any Force according to its proper Direction, is proportionable to the Diſtance of the Line of Direction from the Fulcrum.

THEOR. XI.

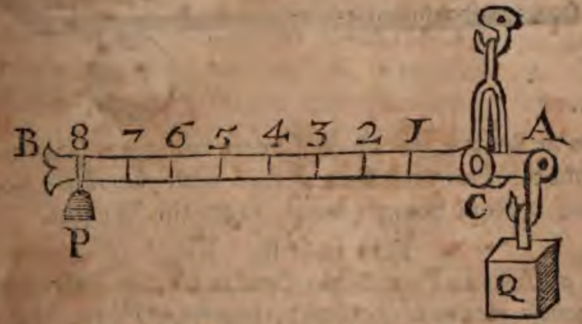
IN *a Lever, the moving Force or Power that has to the Weight the ſame Ratio, which the Diſtance of the Line of Direction of the Weight from the Fulcrum, has to the Diſtance of the Direction of the Power from the Fulcrum, will ſuſtain the Weight; and therefore if it be ever ſo little increaſed, it will raiſe the Weight.*

IT is manifeſt from the preceding Theorem, that the Spaces which are deſcribed by a Power and Weight according or contrary to their proper Propenſions, are proportionable to the Diſtances of the Line of Directions from the Fulcrum; but the Velocities are proportionable to theſe Spaces, and conſequently will be alſo proportionable to the Diſtances. If therefore the Power P is to the Weight Q as CQ, the Diſtance of the Direction of the Weight from the Fulcrum to CA, the Diſtance of the Direction of the Power from the Fulcrum, the Power will be to the Weight, as the Velocity of the Weight to the Velocity of the Power; the Momentum therefore of the Power [by Cor. 3. Theor. 3.] will be equal to the Momentum of the Weight: and conſequently, the

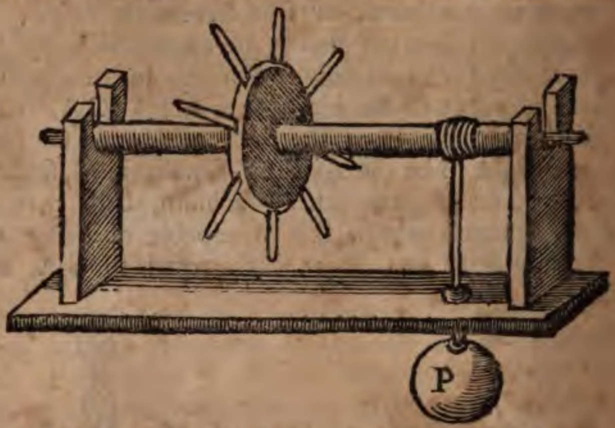


the Power will be equivalent to the Weight; which if it be ever ſo little increaſed, it will raiſe the Weight. *Q. E. D.*

HENCE appears the Reaſon, why by the *Statera Romana* or Steel-yard, as it is commonly called, the Weights of different Bodies are examined all by one and the ſame Weight only. For this Inſtrument is a Lever of unequal Brachia, one whereof, C A, is extended in length, from the Axis of Motion C, (and which ought to be the Axis of Equilibrium) ſuppoſe one Inch, or leſs; the other Brachium, C B, may be of any greater length, that is capable of being exactly divided into Parts, each equal to C A, and numbered by the Figures 1, 2, 3, 4, 5, &c. Let the Body whoſe Weight Q we want to diſcover, be hanged on at A; the given or known Weight P is moveable on the contrary Brachium, and by removing it from or bringing it nearer the Center C, is diſcovered the diſtance where is an exact Equilibrium: as, for example, if the Weight P at the Diſtance 8 equiponderates the Weight Q in A, it is thence concluded (by reaſon the Weights are reciprocally proportionable to their Diſtances) that the Weight Q is eight times the known Weight P.



Defin. THERE is a mechanical Inſtrument proper to raiſe Weights, called an *Axis in Peritrochio.* It is compoſed of a Cylinder, that is named the *Axis*, ſupported at each end by *Fulcra*. About this Cylinder is fixed a Wheel, (which they call *Peritrochium*) and in its Circumference are faſtened ſeveral Handles or Spokes, to which a Force being applied, turns about the *Peritrochium* together with the *Axis*; about which a Rope being wound, raiſes the Weight.



THEOR. XII.

IN *an* Axis in Peritrochio *(and in the like Inſtruments, whoſe Reaſon is the ſame) the moving Force, that has to the Weight to be ſuſtained the ſame Ratio, as the Circumference of the Axis to which that Weight is applied, has to the Circumference of the outer Orb to which the Power is applied, will be equivalent to the Weight; which therefore being ever ſo little increaſed, will raiſe the Weight.*

IT is manifeſt from the Make of the Inſtrument, that in one Converſion the affixed Weight P will be raiſed to an height that is equal in length to as much of the drawing Rope, as will once encompaſs the Axis; which therefore is ſuppoſed to be equal to its Circumference: and in the mean while the Power applied to the Extremity of one of the Spokes, will have proceeded through the Circumference of the outer Orb, deſcribed by the Power in the ſame Revolution of the Inſtrument; (that is, the Space paſſed over by the Power in the ſame Time, will be equal to the Circumference of the outer Orb,) ſo that the Velocities of the Power and of the Weight, which are as the Spaces paſſed over in the ſame Time, will be as the Circumference of the outer Orb, and the Circumference of the Axis. Wherefore if the Weight is to the Power, as the Circumference of the outer Orb is to the Circumference of the Axis, the Velocity of the Power will be reciprocally to the Velocity of the Weight, as the Power to the Weight. Therefore [by Corol. 3. Theor. 3.] the Momentum of the Power will be equal to the Momentum of the Weight; and conſequently, the Power will be equivalent to the Weight, and that Weight will be ſuſtained by the *Axis in Peritrochio:* but if the Power be increaſed, or the Weight leſſened ever ſo little, the Power will raiſe the Weight. *Q. E. D.*

Cor. BY how much the greater is the Compaſs of the outer Orb, that is, by how much the longer the Handles or Spokes are, or by how much the leſs the Axis is, by ſo much the more powerful will be the Force to raiſe the Weight.

Defin. AN Inſtrument compoſed of one or more Wheels aptly diſpoſed, moveable about their Axes, and about which Wheels a Rope being put, draws up the Weight, is called a Pulley.

THEOR. XIII.

IN *a moveable Pulley, from the Poſition of the Wheels may be computed what the Force ought to be, which will be equivalent to an affixed Weight; namely, that Force, which is to the Weight, as* 1 *to the Number of Ropes, by which the Weight is ſuſpended, will be able to ſuſtain that Weight: which Force being therefore ever ſo little increaſed, will raiſe the ſame Weight.*

Fig. 1. LET one of the Ends of a Rope be faſtene to a Hook B, and in its Duplicature let be hung moveable Pulley, to whoſe Block let be fixed th Weight Q; it is manifeſt, that in order to raiſe th

Fig. 1.

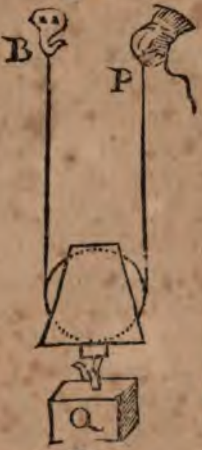


Fig. 2.

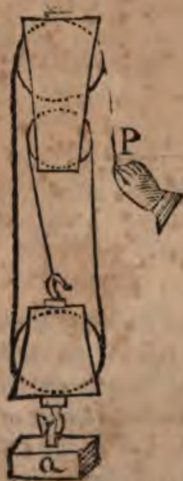
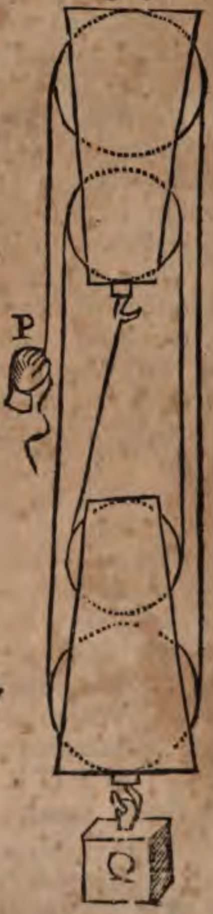


Fig. 3.



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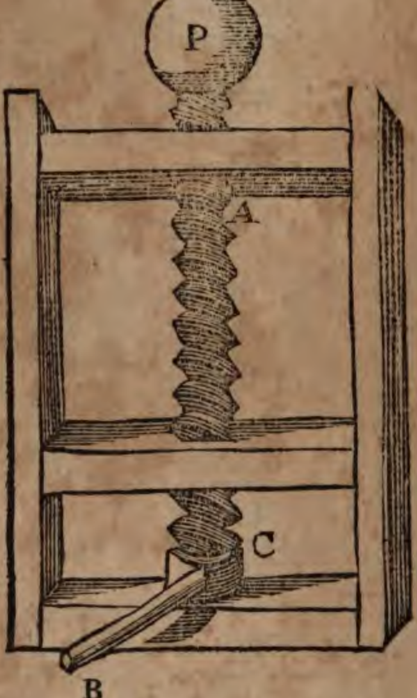
Weight Q one Foot, each Rope ſuſtaining the Block together with the Weight, (reckoning downwards from the Hook) ought to be ſhorten'd alſo one Foot: that is, to raiſe the Weight one Foot, the Power muſt move through two Feet; whence in this Inſtrument, the Way of the Power will be double that of the Weight; and therefore the Celerity of the Power will be likewiſe double that of the Weight: ſo that if the Power is to the Weight as 1 to 2, its Momentum will be equivalent to the Momentum of the Weight, and it will ſuſtain the Weight.

Fig. 2. If the Wheels are ſo diſpoſed, that the Weight Q hangs from three Ropes; and that the Weight may aſcend one Foot, all the three Ropes (we call them three, tho' indeed they are but the Continuation of one uninterrupted Rope) muſt be ſhortened one Foot each, which can be done no otherwiſe, than by the Power P being moved through the Space of three Feet. Whence ſince in this Inſtrument the Way of the Power is triple that of the Weight, its Celerity will be triple the Celerity of the Weight; ſo that if the Power is to the Weight as 1 to 3, its Momentum will be equivalent to the Momentum of the Weight.

Fig. 3. By the ſame way of reaſoning, from the third Figure it is manifeſt, the Power P, which is but the fourth of the Weight Q, will be equivalent to it. In all Caſes, the Power that is equivalent to the Weight, if it be ever ſo little increaſed, or the Weight diminiſhed, the Power will then raiſe the Weight. *Q. E. D.*

Defin. A Right Cylinder indented equally in a helical Form, is called a *Skrew*, whereof there are two ſorts, *Male* and *Female*. It is named a male Skrew, when the convex Surface of the Cylinder is cut in; and a female Skrew, when the concave. The firſt is alſo called the outſide Skrew, as the other is called the inſide one. But the female Skrew ought to be ſo fitted to the male, that their Parts may anſwer to each other, (*viz.* that the Eminences of the one do agree with the Cavities of the other) whereby, if the female Skrew is fixed, the male may be moved quite through it; or if the male Skrew is fixed, the female may be freely moved over it. Skrews are chiefly employed to remove Obſtacles, to break in pieces or to compreſs any thing, and to effect other Motions that are to be produced by Pulſion: the Power is applied to an Handle or Handſpike.



THEOR.

THEOR. XIV.

IN *the Skrew, if it be as the Compaſs, which the Force or Power applied, paſſes over in one Revolution of the Skrew, to the Diſtance between two helical Threds that are contiguous, (reckoned according to the Length of the Skrew) ſo is the Weight or Reſiſtance to the Power; then the Power will be equivalent to the Weight, and that Power ever ſo little increaſed, will move the Impediment.*

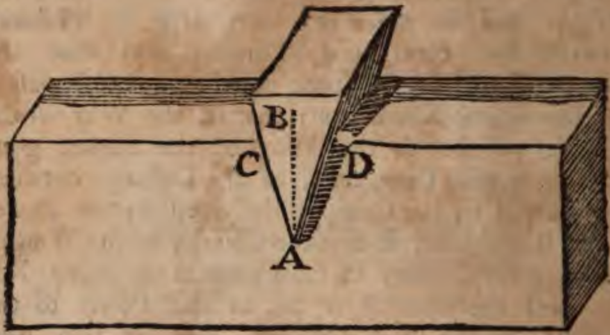
LET CA be a male Skrew, which being turned round by means of the Handle CB, may be paſſed through a female Skrew, and at the ſame time raiſe a Weight P. It is manifeſt by inſpecting the Inſtrument, that in one Revolution of the Skrew, the Weight will be raiſed as much, as is the Diſtance between two contiguous Threds; and that the Power will paſs over a Space equal to the Compaſs it deſcribes in one Revolution: that is, the Way of the Weight will be to the Way of the Power gone through in the ſame time, as the Diſtance between two Threds to the Compaſs deſcribed by the Power in one Revolution, ſo that the Celerity of the Weight will be to the Celerity of the Power in the ſame Ratio: and therefore if it be, as the Power to the Weight, ſo the ſaid Diſtance between two near Threds to the Way deſcribed by the Power, then the Power will be equivalent to the Weight; which being ever ſo little increaſed, it will overcome the Reſiſtance. *Q. E. D.*

Defin. A WEDGE made of Iron, or ſome hard Subſtance, is frequently in uſe. Its form is that of a Priſm of a ſmall Height, whoſe oppoſite Baſes are Iſoſceles Triangles; the Height of theſe Triangles is the Height of the Wedge, the Baſe of the Triangle is alſo called the Thickneſs of the Wedge, and the right Line which joins the Vertices of the Triangles, is the Edge of the Wedge; and the Parallelogram that joins the Baſes of the Triangles, is called the Back of the Wedge.

THEOR. XV.

A POWER *applied directly to the Back of a Wedge, which is to the Reſiſtance that is to be overcome by that Wedge, as the Thickneſs of half the Wedge to its Height, will be equivalent to the Reſiſtance; and therefore if it be increaſed, it will overcome that Reſiſtance.*

LET the Reſiſtance to be overcome by a Wedge be, for example, the Tenacity or Toughneſs of Wood, or any other Obſtacle that is to be cleaved by a Wedge; it is manifeſt, that when the Wedge is driven in as far as it appears in the Figure, the Way of the Power, or the Length run through according to its Propenſity is B A; for ſo far, and no farther, is it gone. And after the ſame manner, the Way of each Impediment is half C D, and when the Wedge is drove through its whole Height, the Obſtacle is divided by the whole Breadth of the Wedge; and conſequently each part of the Obſtacle is moved half the Breadth of the Wedge; and proportionably in its whole Progreſs, as appears from the Nature of

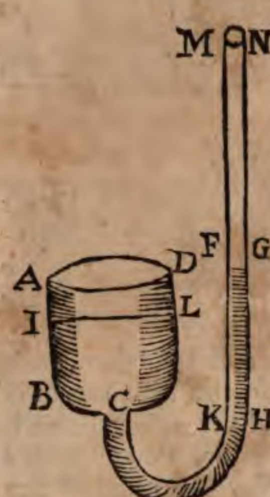


a Triangle. Whence, if the Power be to the Reſiſtance on each ſide the Wedge, as half its Thickneſs to its Height, the Reſiſtances will counterbalance the Power; ſo that if the Power be increaſed, it will overcome the Reſiſtance.

SCHOLIUM.

HENCE it appears, that the Force of the Power is not increaſed by the means of mechanical Inſtruments, which is impoſſible to be done; but by the application of the Inſtrument, the Velocity of the Weight that is either to be raiſed or drawn along, is diminiſhed in ſuch a manner, that the Momentum of the Weight becomes leſs than the Momentum of the Power. So, for example, if any acting Force is able to raiſe a given Weight of one Pound with a given Velocity, it is impoſſible, by the means of any Inſtrument whatever, that the ſame Force ſhould be made to raiſe a Weight of two Pounds with the ſame Velocity; yet it may, by the help of an Inſtrument, be able to raiſe a Weight of two Pounds with half that Velocity; nay, the ſame Power will raiſe a Weight of a thouſand, or even ten thouſand Pounds, with the thouſandth or ten thouſandth part of that Velocity; but the Force of the Power is not thereby increaſed: the Motion only that it produces in raiſing that great Weight, is equal to the Motion that is produced when a Weight of one Pound is raiſed.

FROM what has been ſaid, the Reaſon is plain, why in Canals of different Capacities, which communicate with one another, the Equilibrium of Liquors is preſerved. For let A B C D be a large Canal communicating at C with a ſmaller one M N K H; if Water be poured in, it will riſe in both Canals to the ſame height, and the Endeavour to deſcend, or the Force that the Water in the Canal F H has to flow through the Orifice C, is equal to the Force of the



Water in the Canal A C to deſcend through the ſame Orifice. For if we ſuppoſe, that the Water has deſcended in the Canal A C through the Height A I; it is neceſſary, that the Water in the Canal F H do riſe to the Height H N, namely, to ſuch a Height, that the Cylinder of Water M F G N be equal to the Cylinder A I L D, *viz.* the Cylinder of Water, which deſcends in the Canal A C: but in equal Cylinders, the Baſes and Heights are reciprocal [by 15 *El.* 12.] that is, it will be F M to A I, as the Orifice A D to the Orifice M N or F G; but it is F M to A I, as the Velocity of the Aſcent of the Water in the Canal F N, to the Velocity of the Deſcent of the Water in the Canal A C; and the Orifice A D is to the Orifice M N, as the Water in A C to the Water in the Canal F H, for Cylinders of the ſame height are as their Baſes. Whence the Velocity of the Water aſcending in the Canal F H, will be to the Velocity of the Water deſcending in the Canal A C, as the Water in the Canal A C to the Water in F H; that is, the Velocities of the Waters are reciprocally proportionable to the Quantity of the Waters themſelves, and therefore the Momenta of the Waters will be equal: but they are contrary, whence there will follow no Motion.

HENCE, by the bye, appears the Reaſon why Water, or indeed any Fluid, flowing out of a larger into a narrower Channel, is moved with a greater Celerity.

HENCE in an Animal Body, if the ſmall Branches of the Arteries, or the Capillary Arteries, have the Sum of their Orifices, or rather of their tranſverſe Sections, greater than the tranſverſe Section of the great Artery or Aorta, from which they all ariſe; the Velocity of the Blood will be leſs in the Extremities of the Body, than in the Aorta: but if this Sum be equal to the tranſverſe Section of the Aorta, the Velocity of the Blood in the Extremities will be equal to the Velocity of the Blood in the Aorta; if the Sum is leſs, the Velocity of the Blood circulating through the extreme Arteries, will be greater than in the Aorta.

LECTURE XI.

Of the Laws of Nature.

WE have hitherto given the Demonſtrations of Theorems relating to the Quantity of Motion, to Spaces paſſed over by moving Bodies, as likewiſe the Corollaries that might be deduced from them; now then we ſhall proceed to the Laws of Nature; namely, ſuch Laws, as it is neceſſary that all natural Bodies do obey. And we ſhall deliver theſe in the ſame Order, and in the very ſame Words, as they are laid down by the Illuſtrious Sir *Iſaac Newton*; the firſt whereof is this:

LAW I.

EVERY *Body will continue in its State of Reſt, or of moving uniformly in a right Line, except ſo far as it is compelled to change that State by Forces impreſſed.*

SINCE natural Bodies conſiſt of a Maſs of Matter, that of itſelf is not able to induce any Change in its State; if Bodies were once at Reſt, it is neceſſary that they ſhould always remain in that State of Reſt, unleſs there is applied a new Force to produce Motion in them: but if they were in Motion, the ſame Energy or Force would always preſerve the Motion; and therefore Bodies would always retain their Motion, and would always proceed forward in the ſame right Line with the ſame Tenour, ſince they cannot of themſelves acquire either Reſt, or a Retardation, or a Change of their Direction to turn on one ſide or the other. There are ſome Philoſophers, who readily enough acknowledge that no Body can move of itſelf, that is, paſs from Reſt to Motion of itſelf; but then they are not as willing to grant, that Bodies once moved cannot of themſelves arrive at Reſt, by reaſon they ſee the Motions of Projectiles languiſh by degrees, and at laſt the moving Bodies themſelves come to Reſt.

BUT as no Mode, or Accident, can of its own accord or by itſelf be deſtroyed, and as all Effects produced by tranſient Cauſes do remain always, unleſs there be ſome new and extraneous Cauſe that deſtroys them; ſo likewiſe Motion once commenced will be continued always, unleſs it is hindered by ſome external Cauſe: nor is it more in the power of a Body once moved, to lay aſide its Motion or Energy to move, and return of itſelf to Reſt, than it can put off the Figure that it has been once formed into, and acquire a new one, without ſome extrinſick Cauſe.

THERE is beſides in all Bodies a certain Force, or rather Inactivity, whereby they oppoſe every Change; whence it is, that they are very difficultly put out of their State, whatever it is: but that Force is the ſame in moving Bodies, as in thoſe at

Reſt, nor do Bodies leſs reſiſt the Action, whereby they are brought from Motion to Reſt, than that whereby they paſs from Reſt to Motion; that is, there is not required a leſs Force to put a ſtop to the Motion of any Body, than was before neceſſary to impreſs that Motion on the ſame Body. Whence, ſince the *Vis Inertiæ* or Inactivity of Matter always equally reſiſts equal Changes, it will not be leſs powerful to continue a Body in Motion, which has begun to move, than to preſerve a quieſcent Body in the ſame State of Reſt.

THERE are ſome Philoſophers who ſuppoſe Body of its own nature to be as indifferent to Motion as to Reſt; but by this Indifference they do not, I believe, mean ſuch a Diſpoſition in Bodies, whereby they do not in the leaſt reſiſt Reſt or Motion: for on this Suppoſition it would follow, that any Body, however great, and moved with the ſwifteſt Velocity, might be ſtopped by any the ſmalleſt Force; or if the great Body was at reſt, it might be moved by any Body, however ſmall, without the leaſt Loſs of Velocity in the impelling Body: that is, any ſmall Body impinging on a greater one, would carry that greater Body along with it, without the leaſt Loſs of its Motion; and each Body after the Impulſe would be jointly carried along with that Celerity, that the ſmall Body had at firſt, which, we all know, is abſurd. This Indifference therefore is not placed in a Non-reſiſtance to Motion from a State of Reſt, or to Reſt from a State of Motion; but in this only, that a Body of its own nature is not more propenſe to Motion than to Reſt, nor more reſiſts to paſs from a State of Reſt to Motion, than to return again from that Motion to the ſame State of Reſt: beſides, any quieſcent Body may be moved by any Force, and an equal Force acting in a contrary Direction, will be able to deſtroy that Motion; and in this, they would have this Indifference placed.

SINCE, according to this Law of Nature, a Body once in Motion always continues in that Motion, the Philoſophers aſk why all Projectiles loſe by degrees their Motion? (which Motion they are pleaſed to call violent.) Why do they not proceed *in infinitum?* If Motion did not of its own nature decay, a Stone thrown at the beginning of the World would by this time have gone through an immenſe and almoſt infinite Space. And ſo indeed it would, if its Motion had been *in vacuo* or in free Spaces, and without any Gravity. But ſince all Projectiles are carried either through the Air, or on the rough Surfaces of other Bodies, they muſt be neceſſarily retarded: for ſince all Bodies in motion muſt drive and thruſt out of its place the reſiſting Air, or overcome the Roughneſs of the Superficies upon which they are moved; they will loſe all that Force and Motion, that is conſtantly employed in overcoming theſe Obſtacles, and conſequently the Motion of Projectiles will be continually diminiſhed: but if there was no Reſiſtance in the Medium, no Roughneſs in the Superficies on which they were moved, no Gravity that continually forces the Bodies towards the Earth, Motion would always continue the ſame, without any Retardation at all. So in the Heavens, where the Medium is exceedingly rare, the Planets do continue in their Motions for a very long time; and upon Ice, or any other very ſmooth Surface without any Roughneſs, heavy Bodies in motion are not ſoon brought to reſt.

LET the Philoſophers at length leave off inquiring into the Cauſe of the Continuation of Motion; for there is no other beſides that firſt Cauſe, which does not only preſerve Motion, but every thing elſe in its *Being*, namely, the All-wiſe and Great God. Nor is Motion continued any other way, than that whereby by is preſerved the Figure, the Colour, and any other the like Affections of Body, which always remain the ſame, unleſs they are altered by any external Force.

THESE Philoſophers would have done much better, and more ſuitable to the Rules of good Method, if they had inquired out the Reaſons of the Decay or Loſs of Motion: but there are ſome ſo blind in this Affair, that they ſuppoſe that very thing to be the Cauſe of the Continuation of Motion, from which indeed proceeds its Decay.

LET the Philoſophers alſo leave off diſputing concerning the Communication of Motion; for from what has been ſaid, may be underſtood why a Stone goes with ſo great an Impetus out of the Hand of one that throws it; namely, when the Stone is held in the Hand, it is neceſſary that it partakes of the Motion of that Hand [by Axiom 8.] ſo that it will be carried in the ſame Direction with the Hand: but every natural Body once moved, continues in the ſame Motion [by the above-mentioned Law] until it is hindered by ſome external Agent, whence when the Perſon that caſts the Stone draws back his Hand, but not drawing back the Stone, it will proceed right forwards. After the ſame manner, if a Ship or Boat is ſwiftly driven by the Winds or Oars, thoſe that ſit in her, have the ſame ſwift Motion communicated to them; but if the Ship ſtop of a ſudden, every thing that is placed in that Ship endeavours to continue its Motion, and ſuch things as are not firmly fixed to the Ship, do after the Ship is at reſt, leaving their places, ſtill move on; and hence there is a danger, leſt Men relatively at reſt in a Ship, after ſo ſudden and as it were violent Change of its State, are not thrown along; namely, ſince the Motion which they before received from the Ship, is not yet deſtroyed.

IF a Stone is ſwiftly whirled about in a Sling, it deſcribes a Circle with the ſame Velocity as the bottom of the Sling has, wherein the Stone is placed: but ſince every Body affects to proceed in a right Line, the Stone in every Point of its Orb would go out in a right Line, touching that Orb in the Point wherein it is placed, if it was not hindered by the String: ſo that if the String be let go, broke, or any other ways deſiſts from detaining the Stone, the Stone would no longer move in a Circle, but in a right Line, excluding the Motion ariſing from its Gravity.

THAT Endeavour, which the Stone thus whirled about has in any Point of its Orb, of running out in the Tangent, ſtretches a String by which it is detained in that Orb; and that Force, whereby the String is ſtretched, ariſes from the centrifugal Force, whereby, namely, it endeavours to recede from the Periphery. This Tenſion any one may eaſily experience in a Sling; and it is found by Obſervation, that the ſwifter the Stone is whirled about, or the greater the Weight is that is put into the Sling, the greater will be the Tenſion of the String, or it will be ſtretched the more.

FOR this reaſon, ſome Philoſophers will have this centrifugal Force to proceed from Gravity alone, but neither Reaſon nor Experience favours this Opinion; for in a Sling, the String is not only ſtretched when the Stone moves through the loweſt part of its Orb, but likewiſe when it deſcribes the upper part: which cannot ariſe from Gravity, ſince Gravity in the upper part of its Orb can only draw the Stone towards the Center, which is directly contrary to the centrifugal Force, that compels it to recede from the Center. Beſides, if the Stone is whirled about in a Circle parallel to the Plane of the Horizon, the String is then alſo ſtretched; but Gravity can no ways produce that Tenſion, ſince the Stone is neither carried upwards or downwards, whoſe Motion can therefore be neither increaſed or diminiſhed by this Gravity: the centrifugal Force conſequently does not ariſe from Gravity, but only from that Endeavour, which all Bodies have of proceeding in the ſame right Line.

IF we ſuppoſe the Earth to revolve about its Axis, all of us who live upon its Surface muſt revolve together with it; ſo that if its Motion ſhould ſuddenly ceaſe, all things not firmly fixed to it, would be thrown from it with a violent Motion. So likewiſe if it is carried with an annual Motion about the Sun, and that Revolution ſhould be ſuddenly ſtopped, all things would be thrown from it, and, like Planets, would revolve about the Sun, by the ſame Cauſe, as the Earth itſelf was moved before about the Sun.

SINCE the Earth turns about its Axis, and all things on it deſcribe Circles parallel to the Plane of the Equator, the Philoſophers demand how it comes to paſs, that all Bodies are not thrown off from its Surface, when by the Law of Nature all Bodies affect a Motion in a right Line? So indeed they would be thrown off, if they were not detained on the Earth by another Force, which is Gravity, and which is much more potent than the centrifugal Force.

IF a Veſſel full of Water is placed on any horizontal Plane, and ſuddenly puſhed forward by any great Force, the Water in the Veſſel will at the beginning be ſeen to tend towards the Parts that are contrary to the Motion of the Veſſel: not that in reality ſuch a Motion is impreſſed on the Water, but ſince it endeavours to continue in the ſame State of Reſt, the Veſſel cannot immediately communicate its Motion to the Water contained in it; and therefore the Water deſerted by the Veſſel, and truly at reſt, will ſeem to change its relative place. But at length after the Motion of the Veſſel is communicated to the Water, and that together with the Veſſel begins to move uniformly and with the ſame Celerity, if then the Veſſel be ſuddenly ſtopped, yet the Water will endeavour to continue in the ſame Motion, and riſing up the Sides of the Veſſel, part of it will even run over.

If a Ship is toſſed by a Storm and a troubled Sea, the Men that are ſitting in her, and relatively at reſt, will be ſeized with Pains, Sickneſs, a Nauſea and Vomiting, eſpecially if they have not been uſed to the Sea; ſince the Liquors contained in their Stomachs, Inteſtines, Blood-Veſſels and other Ducts, do not immediately obſerve the Toſſings of the Ship, whence the Motion of the Fluids in the human Body is diſturbed, and Diſeaſes do ariſe.

LAW II.

The *Change of Motion is always proportionable to the moving Force impreſſed, and is always made according to the right Line, in which that Force is impreſſed.*

This follows from the fourth Axiom; for if any Force generates a certain Motion, a double Force will generate a double Motion, and a triple one a triple Motion: and this Motion, becauſe it is determined always into the ſame Direction with the moving Force, (for it ſolely ariſes from it) it will be always made according to the ſame Direction [by the firſt Law,] nor can the Body decline according to any other Direction, unleſs ſome new Force oppoſes the former; ſo that if the Body was moving before, the Motion produced from the impreſſed Force will either be added to the conſpiring Motion, or ſubducted from the oppoſing Motion, or obliquely joined to the oblique Motion, and will be compounded with it according to the Determination of both.

IF any Force produces a Motion in a given Body, [by the firſt Law] that Body will always continue in its Motion: but if afterwards the ſame or an equal Force again acts on that ſame Body according to the ſame Direction, the Motion thence ariſing will be equal to the former, and therefore the Sum of the Motions will be double that former; if the ſame Force a third time acts in like manner on the ſame Body, the Motion hence occaſioned will be alſo equal to the firſt, and therefore the Sum of the Motions will be triple the Motion firſt impreſſed: and if the ſame Force ſhould act again on the ſame Body as before, the Sum of all the Motions will be quadruple the Motion firſt impreſſed, and ſo on continually.

HENCE if this new Force ſhould act continually and equally, in equal Intervals of Times, the Motion thence ariſing would be as the Sum of the Times in which it was generated: ſo that, ſince by reaſon the Body is given, the Motion is as the Velocity, the Velocities ſo generated will be as the Times from the beginning of the Motion, and the Motion will be equally accelerated. Hence the following Theorems may be eaſily demonſtrated.

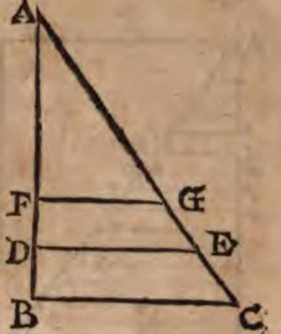
THEOR. XVI.

IF *Bodies at all Diſtances from the Earth gravitate equally, the Motion of Bodies falling by their Gravity in the ſame right Line, will be an equably accelerated Motion.*

LET the Time in which the Body is falling, be ſuppoſed to be divided into equal and very ſmall Particles, and let the Gravity acting in the firſt Particle of Time draw the Body towards the Center; if now after that firſt Time the Action of Gravity ſhould ceaſe, and the Body be no longer heavy, nevertheleſs the Motion received from the firſt Impulſe would be always continued, and the Body would equally deſcend to the Earth, [by the firſt Law;] but ſince the Body is conſtantly heavy, and Gravity acts without intermiſſion, alſo in the ſecond Particle of Time the ſame Gravitation will communicate to it another Impulſe equal to the former, and the Velocity of the Body after theſe two Impulſes will be double the former Velocity; and if the Force of Gravity was quite removed, yet the Body would continue to move in the ſame right Line with the ſame Velocity: but ſince in the third Particle of Time the Body is acted upon by the ſame Gravity, it will alſo after that third Time acquire another Motion equal to each of the former; ſo likewiſe in the fourth Particle of Time, the Gravitation will add to the heavy Body a fourth Impetus equal to each of the former: and ſo of the reſt. The Impetus therefore or Motions of a given Body acquired by its Gravity, are as the Particles of the Time elapſed from the beginning: ſo that ſince the Action of Gravitation is continual, if thoſe Particles are taken infinitely ſmall, the Motion of the falling Body acquired by its Gravity, will be as the Time elapſed from the beginning of the Fall; and ſince the Body is given, its Motion will be as its Velocity, therefore the Velocity will be always as the Time in which it was acquired. Inſomuch that in equal Intervals of Times, a falling Body will receive an equal Increaſe in its Velocity, and conſequently its Motion will be uniformly accelerated. *Q. E. D.*

In like manner, from the ſame Principles may be demonſtrated, that the Motion of Bodies thrown upwards is equably retarded; ſince the Force of Gravity, acting conſtantly and equally againſt the Motion commenced, in equal Times will equally diminiſh its Motion, even till all the Motion upwards is quite deſtroyed.

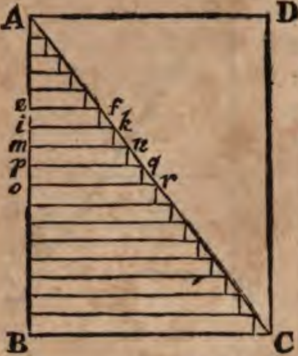
Cor. Let the right Line A B repreſent the Time in which a Body is falling, and B C making with A B a right Angle, repreſent the Velocity acquired at the end of the Fall; join A C, and through any Point D let be drawn D E parallel to B C: this will be as the Velocity acquired at the end of the Time A D. For (by reaſon of the ſimilar Triangles A B C, A D E) A B is to A D as B C to D E; but B C repreſents the Velocity acquired in the Time A B, whence (ſince the Velocities are as the Times) D E will repreſent the Velocity acquired at the end of the Time A D: in like manner, F G will repreſent the Velocity in the Point of Time F, and in every Point of Time the Velocities will be as the right Lines drawn within the Triangle through it, and parallel to the Baſe B C.



THEOR. XVII.

If *a heavy Body deſcends from Reſt with a Motion uniformly accelerated, the Space, that from the beginning of its Motion is run over by the Body in a given Time, will be half the Space that may be uniformly run over in that Time, with a Velocity equal to that which the heavy falling Body will have acquired at the end of that Time.*

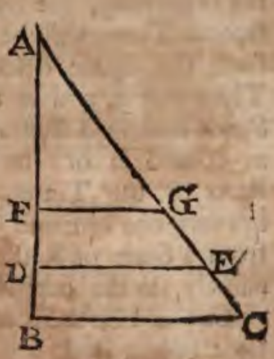
Let A B be the Time in which the heavy Body is a falling, and let B C be the Velocity acquired at the end of the Fall; compleat the Triangle A B C, and Rectangle A B C D; moreover, let the Time A B be diſtinguiſhed into innumerable Particles, *e i*, *i m*, *m p*, &c. draw *e f*, *i k*, *m n*, *p q*, &c. parallel to the



Baſe; [by the preceding Corol.] *ef* will be as the Velocity of the heavy Body in the infinitely ſmall Particle of Time *ei*; and *ik* will be its Velocity in the Particle of Time *im*; alſo *mn* will be its Velocity at the Point of Time *mp*; and ſo *pq* will be the Velocity in the Particle of Time *po*: but [by Cor. Theor. 7.] the Space paſſed over in any Time, and with any Velocity, is a Rectangle under that Time and Velocity: whence the Space paſſed over in the Time *ei* with the Velocity *ef*, will be as the Rectangle *if*; ſo the Space paſſed over in the Time *im* with the Velocity *ik*, will be as the Rectangle *mk*; ſo likewiſe the Space paſſed over in the Time *mp* with the Celerity *mn*, will be as the Rectangle *pn*: and ſo of the reſt. Wherefore the Space paſſed over in all theſe Times, will be as all theſe Rectangles, or as the Sum of all theſe Rectangles; but ſince the Particles of Time are infinitely ſmall, the Sum of all the Rectangles will be equal to the Triangle A B C: but [by the Corol. of Theor. 7.] the Space paſſed over by a moving Body in the Time A B with an uniform Celerity B C, is as the Rectangle A B C D. Whence the Space paſſed over in a given Time by a heavy Body falling from Reſt, will be to the Space paſſed over in the ſame Time with an uniform Velocity, equal to that which is acquired by the heavy Body at the end of its Fall, as the Triangle A B C to the Rectangle A B C D; but the Triangle A B C is half the Rectangle A B C D, whence the Space that is paſſed over in a given Time by a falling Body from the beginning of its Fall, will be half that Space which it would have paſſed over in the ſame time, with an uniform Velocity equal to that which it has acquired at the end of its fall. *Q. E. D.*

Cor. 1. THE Space which is paſſed over with the Velocity CB in a Time equal to half AB, will be equal to the Space paſſed over by a falling heavy Body in the Time AB.

Cor. 2. FROM the Demonſtration itſelf it follows, that as the Space paſſed over in the Time AB may be repreſented by the Triangle ABC, ſo the Space deſcribed by the heavy Body in the Time AF may be repreſented by the Triangle AFG: alſo the Space gone through in the Time AD, may be expounded by the Triangle ADE.



Cor. 3. THE Spaces paſſed over, reckoning from the beginning of the Fall, are in a duplicate Ratio of the Times: for the Space paſſed over in the Time AB, is to the Space paſſed over in the Time AF, as the Triangle ABC to the Triangle AFG; but (by reaſon of the ſimilar Triangles ABC, AFG) the Triangle ABC is to the Triangle AFG in a duplicate Ratio of the Side AB to the Side AF: ſo that the Space paſſed over in the Time AB will be to the Space paſſed over in the Time AF in a duplicate Ratio of the Time AB to the Time AF. The Spaces therefore paſſed over by a heavy Body from the beginning of its Fall, are as the Squares of the Times in which they are run through.

Cor. 4. HENCE if a heavy Body falling from a State of Reſt, paſſes over a certain Space in a given Time, the Space paſſed over in a double Time will be quadruple the former Space; and the Space run over in a triple Time will be nine times greater than the Space firſt run over, *&c.* that is, if the Times are taken as 1, 2, 3, 4, 5, *&c.* the Spaces deſcribed in theſe Times reckoning from the beginning of Motion, will be as 1, 4, 9, 16, 25, *&c.*

Cor. 5. SINCE the Space paſſed over in the firſt Time is as 1, in the ſecond as 4, computing from the beginning, the Space deſcribed ſeparately in the ſecond Time will be as 3. After the ſame manner, ſince the Space deſcribed at the end of the third Time is as 9, and at the end of the ſecond as 4, the Space deſcribed ſeparately in the third Time will be as 5; and ſo of the reſt: taking therefore equal Parts of the Times, the Spaces deſcribed ſeparately in every one of thoſe Times by a heavy Body falling from a State of Reſt, will be as 1, 3, 5, 7, 9, 11, *&c.* namely, as the odd Numbers.

Cor. 6. HENCE alſo, ſince the Velocities acquired by a falling Body are as the Times, the Spaces paſſed over will be likewiſe as the Squares of the Velocities: and the Velocities, as well as the Times, will be in a ſubduplicate Ratio of the Spaces through which the heavy Body falls, computing from the beginning of the Motion.

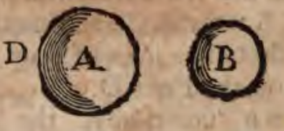
LECTURE XII.

LAW III.

RE-ACTION *is always equal, and contrary to Action; or the Actions of two Bodies upon each other are always equal, and in contrary Directions. That is, by Action and Reaction equal Changes of Motion are produced in Bodies acting upon each other, and theſe Changes are impreſſed towards contrary Parts.*

THIS Law will be better illuſtrated by Examples.

1. IF one Body impinges on another Body at reſt, whatever Motion is impreſſed on the Body at reſt, juſt the ſame Quantity of Motion is ſubſtracted from the impinging Body. For example, if the Body A, whoſe Motion is twelve Parts, is carried towards the Body B, and if after the Impulſe, it ſhall have communicated to B five Parts of its Motion, then will there remain to A but ſeven Parts of Motion, ſo that the Changes which happen to both Bodies will be equal; and the Effect will be exactly the ſame, as if a Force equivalent to five Parts of Motion ſhould puſh the Body B towards C, and another Force equal to the former ſhould act on the Body A, and ſhould preſs it the contrary way towards D.



2. IF the Body B ſhould not be at reſt, but tend towards C, and the Body A moving faſter, ſhould impinge on it; the Body A would loſe juſt the ſame

Quantity of Motion as the Body B would gain, and the Changes of Motion produc'd by the Impulſe on both the Bodies, (that is, the Increaſe of Motion in one, and the Decreaſe in the other) will be equal.

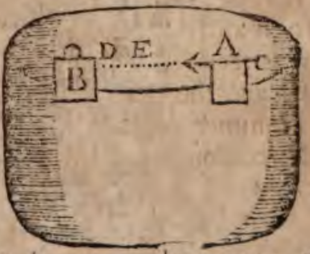
3. If the Bodies A and B meet each other, A being carried towards C with twelve Parts of Motion, but B towards D with three Parts of Motion; whatever Change of Motion happens to the Body B, the very ſame will befall the Body A: for example, if after the Concourſe B is carried towards C with two Parts of Motion, the Change of Motion that is made in it will be five Parts; namely, equal to the Sum of two Motions, *viz.* of that whereby it was before carried towards D, and which was deſtroyed by the Impulſe of the Body A, and of that which it then received, and wherewith it tends towards C; and the Motion loſt in the Body A, will be exactly equal to theſe five Parts of Motion: ſo that (as in the firſt Example) the Effect will follow exactly the ſame, as it would be, if a Force with five Parts of Motion ſhould puſh B towards C, and another Force equal to the former ſhould be impreſſed on the Body A, and preſs it towards the Parts D.

And univerſally, the Magnitude of a Stroke which ariſes from the Concourſe of two Bodies, is always equally receiv'd by both the Bodies; whence the Changes of Motion produced by the Stroke in both the Bodies, will be always equal.

So if an Iron Hammer ſtrikes againſt a Glaſs, the Blow is received equally by the Hammer as by the Glaſs, and the Glaſs is broke, when at the ſame time the Hammer remains whole, not becauſe the Force of Percuſſion impreſſed on the Glaſs is greater than that is which is received by the Hammer, but becauſe the Parts of the Iron being harder, and cohering more firmly together, do more ſtrongly reſiſt the ſame Force of Percuſſion, than the brittle and leſs cohering Parts of the Glaſs can do, Juſt after the ſame manner, if a Body is faſten'd to a Wall by a fine Thred, a ſmall force will be ſufficient to pull it from thence; but if the ſame Body is faſten'd by a ſtrong Rope, the former Force equally applied, will little avail to draw it away.

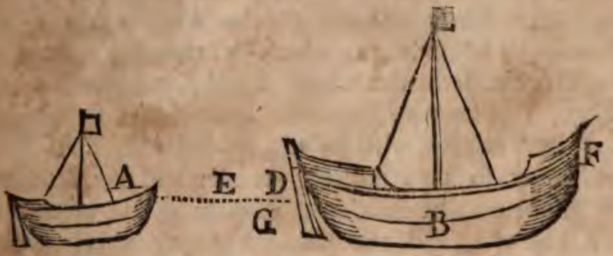
4. If a Horſe draws forwards a Stone tied to a Rope, the Horſe alſo will be equally drawn backwards towards the ſame Stone; for the Rope ſtretched both ways, by the ſame Endeavour of relaxing itſelf, will equally ſtrain the Stone towards the Horſe, and the Horſe towards the Stone: whence the Forces of Attraction, as well in the Horſe as in the Stone, will be equal; but ſince the Firmneſs and Force of the Horſe ſtanding upon the Ground is ſo great, that he is able to reſiſt the drawing of the Rope, he will not in the leaſt yield to the drawing Rope, nor by its Force be removed out of his place; but the Stone, wherein there is not ſo great a reſiſting Force, will be moved towards the Horſe.

5. We find by Experience, that in magnetick Attractions the Loadſtone does not only draw the Iron, but is in like manner equally drawn by the Iron: For let a Loadſtone be placed on a piece of Cork B, and likewiſe the Iron A on another piece of Cork, and let them both ſwim in Water; then let the Loadſtone be held by any one's Hand, and we ſhall ſee the Iron advance to the Loadſtone: and if the Iron is kept immoveable, we ſhall find the Loadſtone will come to it: but if both the Bodies are permitted to float freely on the water, we ſhall behold both the Loadſtone and the Iron mutually



meet each other, and the Force of Attraction acts equally upon them both, producing equal Motions in each: I ſay, the Motions will be equal, but not the Celerities, unleſs the Iron and the Loadſtone weighed alike; for if they are of different Weights, that which weighs the moſt, will have the leaſt Celerity. For example, if the Loadſtone is ten times heavier than the Iron, the Iron reciprocally will have ten times a greater Velocity; namely, that there may be generated equal Quantities of Motion in each Body: ſo that the Loadſtone and the Iron will not meet in the middle Point E, but in the Point D, which will ſo divide the Diſtance B A, as B D will be to D A as the Weight A to the Weight B. So in the preſent Example, if B D is the eleventh part of the whole Diſtance, the Point D will be where the Loadſtone and the Iron will meet each other: for ſince B D is the eleventh part of the Diſtance B A, B D will be to D A as 1 to 10; but as 1 to 10, ſo (by what was ſaid before) will be the Velocity of the Body B to the Velocity of the Body A. Wherefore ſince the Spaces paſſed over in a given Time are proportionable to the Velocities, during the time the Body A is running over the Space A D, the Body B carried with the tenth part of the other's Velocity will have paſſed over a Space equal to the tenth part of the Space run over by the Body A: ſo that at the end of that Time it will be found in D, in which Point therefore the Loadſtone and Iron will meet each other. After the ſame manner, two Loadſtones being placed on two Pieces of Cork, if their correſponding Poles be ſet towards each other, they will mutually attract each other equally; but if their adverſe Poles be placed near one another, theſe Poles will mutually fly from each other, and the Quantities of Motion produced by the Force wherewith they avoid each other, will be equal in both.

6. THE ſame thing may be ſhewn in other kinds of Attractions. For let A and B be two Barges upon the Water, and let a Man placed in one of them, as, for example, in A, draw by the means of a Rope the other Barge B towards himſelf; by this Action of the Man, not only the Barge B will approach towards A, but A alſo will be equally drawn towards B; and the Quantities of the Motions produced by the Attraction, will be equal in each Barge: whence if the Barges are of the ſame Weight, they will have, *cæteris paribus*, equal Velocities, and they will meet at the middle Point E. But if one is greater than the other, that is, has in itſelf a greater Quantity of Matter, or a greater Weight, then that which is the biggeſt will have the leaſt Velocity. For example, if the Barge B is ten times greater than the Barge A, the Velocity of A will exceed the Velocity of the Barge B ten times, and the Barges will meet in the Point G, which ſo divides their firſt Diſtance A D, that A G is ten times greater than G D; that is, G D will be the eleventh part of the whole Diſtance A D: but if B ſhould be a Ship a thouſand or ten thouſand times bigger than A, its Velocity would be a thouſand or ten thouſand times leſs than the Velocity of A, and conſequently it would be ſcarcely diſcernible. Now if B ſhould be another Body infinitely great, its Velocity



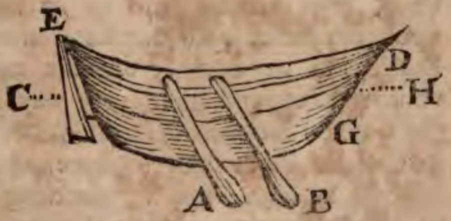
locity would be infinitely ſmall, that is, juſt nothing in reſpect of the Velocity of A. Hence if the Rope ſhould be faſtened to the Shore, and the Man in the Barge ſhould draw by the Rope the Shore towards himſelf, the Barge would approach towards the Shore, and the Shore towards the Barge: but ſince the Shore firmly adheres to the whole Body of the Earth; its Magnitude, which is therefore equal to the Magnitude of the whole Earth, in reſpect to the Barge, will be very immenſe and almoſt infinite, ſo that its Velocity will be nearly infinitely ſmall, and (as I may ſo ſay) nothing at all: and conſequently the Shore may be look'd upon as fix'd and impoſſible to be mov'd, and the whole Velocity as altogether appertaining to the Barge. If the Weight of the Barge B is a thouſand Talents, and is carried towards F with an hundred Degrees of Motion, the Momentum of that Barge will be [by Theor. 3.] an hundred thouſand parts. Now if to the Barge B is faſtened the Boat A, whoſe Weight is ten Talents, whatever Motion by this means is communicated to the Boat A, the very ſame Quantity of Motion will be taken from the Barge B. So that,

7. IF any one in the Boat A draws the Rope A E, by which the Boat is faſtened to the Barge B, in ſuch a manner that the Boat moves forward with five hundred parts of Velocity, the Motion thence ariſing will be five thouſand parts; and ſo much of its Motion the Barge B will loſe, and conſequently there will remain to it ninety-five thouſand Parts of Motion, whence the Velocity of the Barge B will be ninety-five Parts.

8. IF any one ſitting in the Boat A, ſhould by a Spreat, or the like Inſtrument, puſh or thruſt the Barge B towards the Parts F, by that puſhing the Boat A would recede towards the contrary Parts; ſo that the Quantity of Motion in both the Veſſels,

ariſing from the Force of the Man's puſhing, would be equal: whence if the Barge B is ten times greater than the Boat A, it will have ten times a leſs Velocity; if it is an hundred times bigger, it will likewiſe have the hundredth part of the Velocity of the Boat A: ſo that if the Body B is an immenſe Body, the Velocity of the Boat A will be immenſe in reſpect of that which ought to be found in B. Whence if any one ſitting in a Ship, ſhould by means of a Pole, endeavour to thruſt the Earth or the Shore from him, the Ship by his thruſting would recede from the Shore; for the Shore, in reſpect of the Ship may be looked upon as an immenſe and fixed Body, whoſe Velocity therefore will be exceeding ſmall, or indeed none at all, in reſpect to that which is found in the Ship.

If the Boat E D G is rowed by Oars, ſince the Water is by the broad Ends of the Oars A, B, driven backwards towards the Parts C, that will again equally re-act upon the Oars, and force them, together with the Boat to which they are fixed, towards the Parts H: for which reaſon alone it is, that the Boat is moved forwards; for if there was no Re-action, and the Water impreſſed no Motion on the Oars towards the Parts H, when it was thruſt by the Oars the contrary way, the Boat would ſtand ſtill, ſince there would be nothing that could force it towards the Parts H. But ſince the Water by its Re-action impreſſes as great a Motion on the Boat E D, as it receives itſelf from the Oars, it hence follows, the greater



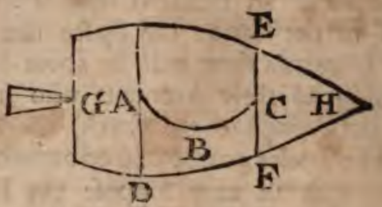
greater the broad Ends of the Oars are, or the more they are in number, *cæteris paribus*, or elſe the ſwifter they are moved within the Water, the Boat will be carried with a greater Impetus.

HENCE ſince ſwimming is nothing elſe than a rowing with the Hands and Feet, it may be eaſily underſtood, why by ſwimming within the Water we are moved forwards; namely, ſince the Water is impelled backwards by the Palms of our Hands and the Soles of our Feet, that by its re-acting forces thoſe that ſwim towards the contrary parts: ſo that the Motion generated in the Water, is equal to the Motion wherewith the Swimmers proceed forwards. It is juſt the ſame thing in the flying of Birds; for as the Birds by their Wings bear the Air downwards, the Air by its Re-action raiſes them upwards; and if they drive the Air towards the Eaſt, for example, the Re-action of the Air obliges them to tend towards the Weſt. So Gunpowder within a Cannon being fired rarefies, and by its Force acts equally upon the Bullet and the Cannon out of which that Bullet is to be hurled; for the rarefied Air endeavouring to expand itſelf every way, will equally preſs the Cannon backwards, as the Bullet forwards, whence its elaſtick Force will produce equal Quantities of Motion in both; and by dividing theſe Quantities of Motion, as well by the Weight of the Engine as by the Weight of the Bullet, the Velocities thence ariſing will be reciprocally proportionable to the Weights.

SINCE all Bodies near the Superficies of the Earth gravitate towards the Earth, in like manner the Earth will gravitate towards each Body, and is attracted towards them; and the Motions generated by this Attraction will be equal, as well in the Earth, as in the heavy Bodies falling to the Earth: ſo if a Stone by the Force of its Gravity falls downwards to the Earth, the Earth in like manner will riſe towards the Stone; but ſince the Quantity of Matter in the Earth exceeds immenſely the Quantity of Matter in the Stone, the Velocity of the Stone will exceed immenſely the Velocity wherewith the Earth tends towards the Stone, ſo that (to ſpeak phyſically) the Velocity of the Earth will be nothing: which will thus appear upon a Calculation. Let us ſuppoſe a Stone of an hundred ſolid Feet in magnitude to fall to the Earth: the Space run over by the Stone in a Minute's time will be about fifteen Feet; but (according to their Accounts who have written concerning the Bigneſs of the Earth) the whole terraqueous Globe contains 30 000 000 000 000 000 000 ſolid Feet. Let us now ſuppoſe the Earth to be every where of the ſame denſity with common Stones, (tho' it is probable it is much more denſe) whence the Quantity of Matter in the Earth will be to the Quantity of Matter in the Stone of a hundred feet, as 300 000 000 000 000 000 to 1; and therefore whilſt the Stone by its Weight falls through the Space of fifteen Feet, the Earth will be drawn towards the Stone the $\frac{15}{300\,000\,000\,000\,000\,000}$ Parts of a Foot: which is ſo very ſmall a Quantity, as not to be conceived by the Imagination, and conſequently may be neglected and eſteemed as nothing in Philoſophy; altho' ſpeaking geometrically, and according to Truth, we ought to ſay that the Earth advanced towards the Stone, and that both the Bodies mutually attracted each other.

If the Moon by its Gravity is retained in its Orb, ſo that it does not leave the Earth; that is, if the Moon gravitates towards the Earth, in like manner the Earth and all its Parts will gravitate towards the Moon: and hence ariſes the continual Flux and Reflux of the Sea. But this by the bye, for we ſhall in another place explain more fully the Motions of the Sea.

LET ſuch a Ship be at reſt in the Water, as may be eaſily moved by any external Impulſe, yet no Force acting within that Ship, and ſolely contained in it, can communicate any Motion to it at all; for let G H be the Ship, and let any Machine, for example, the elaſtick or ſpringing Body A B C, which being violently ſtrained, will endeavour to unbend itſelf, be placed within the Ship: moreover, the Machine being compreſſed, the Side B C will approach the Side A B; the Spring by its natural Energy, or reſtoring Force, endeavouring equally to unbend itſelf on both ſides, will equally impel the Obſtacle D A towards G, and the Obſtacle E F towards H: and therefore the Ship being impelled by theſe contrary and equal Motions, will not be moved at all. Juſt in the ſame manner, as if one ſtanding at the head of the Ship H, ſhould by a Rope draw to himſelf the Stern G; now the Rope being diſtended both ways, by its Endeavour to relax it ſelf, it will equally urge the Stern towards the Man that draws, and the Man towards the Stern; and ſince he is ſeated at the head of the Ship, in like manner the Head will be equally drawn towards the Stern: whence the two contrary and equal Motions will deſtroy each other, and conſequently no Motion at all will follow.

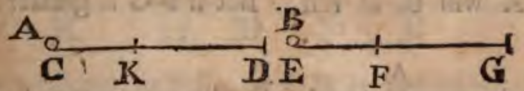


FROM this Law may be demonſtrated the following Theorems.

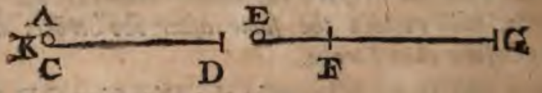
THEOR. XVIII.

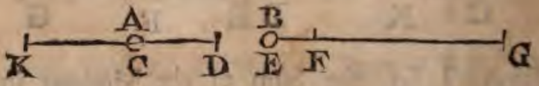
IF *one Body ſtrikes againſt another that is either at reſt, or moving in the ſame Direction, but ſlower; the Sum of the Motions in both the Bodies towards the ſame Parts, will remain the ſame after the Stroke, as it was before that Stroke.*

LET the Body A move in the Direction C D from C towards D, and let it ſtrike againſt another Body B, that is either at reſt, or moves ſlower in the ſame Direction: I ſay, the Sum of the Motions in both Bodies towards the ſame Parts, namely, from C towards D, before and after the Stroke, will remain the ſame. Let CD expreſs the Motion of the Body A, and if the Body B moves, let the right Line EF repreſent its Motion towards the ſame Parts, and therefore the Sum of the Motions may be expreſſed by the Sum of the right Lines CD, EF. Now ſince Action and Re-action are always equal and contrary, the equal Forces impreſſed towards contrary Parts, will produce in both the Bodies equal Changes of Motion towards the contrary Parts: if therefore the Motion impreſſed on B by the Stroke of the Body A ſhall be repreſented by FG, the contrary and equal Force acting on the Body A will take as much from its Motion made towards the ſame Parts; ſo that by putting DK equal to FG, CK will be as the Motion of the Body A, and EG as the Motion of the Body B after the Stroke or Concourſe; and therefore the Sum of the Motions will



will be as the Sum of the right Lines CK, EG. But ſince FG is equal to KD, if to both there be added EF and CK, then EG and CK will be equal to CD, EF: whence the Sum of the Motions towards the ſame Parts, will remain the ſame, both before and after the Stroke. If FG is equal to CD, the Point K will coincide with C, and CK will become equal to nothing; whence after the Stroke the Body A will be at reſt. But if FG is greater than CD, the Point K will fall beyond C, and the Motion of A will become negative, or towards the contrary Parts, *viz.* from C towards K; and the Sum of the Motions towards the Parts G, will be as EG leſſened by CK: for the Sum of two Quantities, whereof one is poſitive, the other negative, is their difference. But becauſe $FC = KD$, let $EF - CK$ be added to them both, and it will be $EF + FG - CK$: that is, $EG - CK = KD + EF - CK$, that is, $EF + CD$: whence the Sum of the Motions towards the ſame Parts, which is here the Difference of the Motions towards the contrary Parts, before and after the Stroke, remains the ſame. *Q. E. D.*





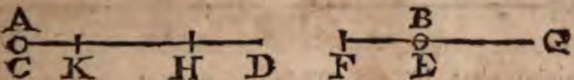
Cor. AFTER the ſame manner, if more Bodies moving towards the ſame Parts ſtrike againſt one another, the Sum of their Motions will not be altered.

THEOR.

THEOR. XIX.

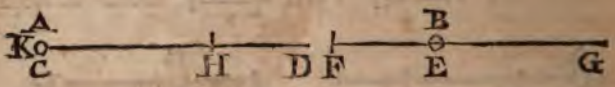
If two Bodies moving towards the contrary Parts, do directly ſtrike againſt each other; the Sum of their Motions towards the ſame Parts (which is the Difference of their Motions towards contrary Parts) before and after the Stroke, will always remain the ſame towards the ſame Parts.

Let the Body A move from C towards D, whoſe Motion let be expreſſed by CD; but let B be moved towards the contrary Part, namely from E to F, with a Motion as EF: let DH be put equal to EF; and CH, which is the Difference of the Motions towards the contrary Parts, will be as the Sum of the Motions towards G: I ſay, the ſame CH will be as the Sum of the Motions towards the ſame Part G after the Stroke. For let the Motion of the Body B after the Stroke be towards the Part G, and let it be repreſented by the right Line EG; the Force therefore of Impulſe impreſſed on the Body B towards the Part G, will be equivalent to the Sum of the Motions EF, EG, and will be repreſented by the right Line FG: for by that Force is deſtroyed the Motion as EF, towards the Part F, and a new one as EG is impreſſed towards the contrary Part G. But ſince the Force of Impulſe acts equally on both Bodies towards the contrary Parts, if DK is made equal to FG, it will repreſent the Force exerciſed on the Body A towards the Part contrary to its Motion; ſo that if the Motion as DK is ſubſtracted from the Motion as CD, there will remain CK as the true Motion of the Body A towards the Part G.

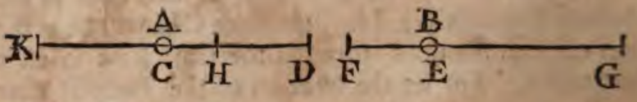


Now ſince D K is equal to F G, and D H equal to F E, it will be D K, leſſened by D H, that is, K H equal to F G, leſſened by F E, that is, E G: and therefore ſince K H is equal to E G, K H will be as the Motion of the Body B after the Stroke; but C K is as the Motion of the Body A, ſo that C K, K H, that is, C H will be the Sum of the Motions in both Bodies towards the Part G. *Q. E. D.*

If F G is equal to C D, the Point K will fall on C, and then the Motion of A will become equal to



nothing; that is, the Body A after the Stroke will be at reſt, and C H will be equal to E G. But if F G is greater than C D, the Point K will fall be-



yond C towards the other Part, and the Motion of the Body A will be from C towards K; but (by reaſon F G is equal to D K, and F E equal to D H) K H is equal to E G: and therefore if from both there be taken C K, it will be C H equal to the right Line E G, leſſened by C K. But C H was as the Sum of the Motions towards the Part G before the Stroke, and it is E G leſſened by C K, as the Sum of the Motions towards the ſame Part; namely, as the Difference of the Motions towards the contrary Parts after the Stroke. Wherefore the Sum of the Motions towards the ſame Part will remain the ſame before and after the Stroke.

These two laſt Theorems are both at once thus well expreſſed by Sir *Iſaac Newton.*

THE *Quantity of Motion, that is gathered by taking the Sum of the Motions made towards the ſame Part, and the Difference of thoſe made towards the contrary Parts, is not altered by the Action of Bodies amongſt themſelves.*

LECTURE XIII.

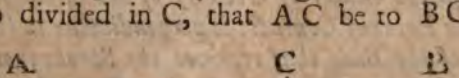
Second Definitions.

I. THE *Center of Gravity of any Body is a Point placed within that Body, through which if a Plane paſſes any how, the Segments of the heavy Body that are on each ſide, as it were ballanced upon that Plane, will equiponderate.*

HENCE, if a Body is ſuſpended by its Center of Gravity, whatever Situation it is put into, it will retain it: *viz.* ſince the Parts of the Body every where about the Center conſiſt of equal Momenta, or have equal Propenſities to Motion.

II. WE *call the common Center of Gravity of two Bodies, a Point ſo ſituated in the right Line joining their Centers, that the Diſtances of the Bodies from that Point, are in a reciprocal Ratio of the Bodies.*

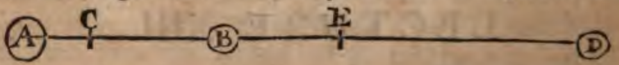
LET there be two Bodies A, B, whoſe Centers of Gravity let be joined by the right Line AB; which let be ſo divided in C, that AC be to BC as the



Body B, that is, the Matter of B is to the Body A, or Matter in A; that Point C is called the common

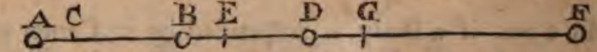
Center of Gravity of the Bodies A and B: namely, becauſe if thoſe Bodies are turned about that Point, at the ſame Diſtances from it, they will retain any given Situation, (as was demonſtrated in Theor. 11.)

III. In *like manner, if there are three Bodies* A, B, D, *and* C *be the Center of Gravity of two of them,* A *and* B, *and the right Line* CD *be ſo divided in* E, *that* CE



is to DE *as the Weight of the Body* D *to the Weight of the two Bodies* A *and* B *together, that Point* E *is called the common Center of Gravity of theſe three Bodies; about which alſo if the Bodies are turned, they will retain any given Situation.*

IV. After *the ſame manner, if there are four Bodies* A, B, D, F, *and* E *is the common Center of Gravity of the three* A, B, D; *the Point* G, *which ſo divides the right Line* EF, *that* EG *is to* GF *as the Weight of the Body* F *to the Weight of the Bodies* A, B, D, *taken together, is called the common Center of Gravity of theſe four Bodies.*



And after the ſame manner the common Center of Gravity of five or more Bodies may be determined.

V. One *Body is ſaid to ſtrike or impinge directly on another, when the right Line, in which it moves, drawn through the Center of Gravity of the impinging Body and the Point of Contact, is perpendicular to the Superficies of the Body that receives the Stroke; or, if the Bodies do not touch one another in a Point, but in a Line or a Superficies, when that Line is perpendicular to this Line or Superficies.*

VI. But *it is ſaid to ſtrike or impinge obliquely or indirectly on the other, when the before-mentioned right Line is not perpendicular to the Superficies of the Body that receives the Stroke.*

VII. I Call *that a perfectly hard Body, which does not yield in the leaſt to a Stroke; that is, which does not loſe its Figure for a Moment.*

VIII. That *is a ſoft Body, which ſo yields to a Stroke, as to loſe its firſt Figure, and never to endeavour to recover it again.*

IX. That *is an elaſtic Body, which yields indeed for a little while to a Stroke, yet of its own accord does recover its firſt Figure.*

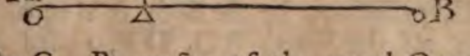
X. An *elaſtick Force is that Force, whereby a Body obliged to quit its Figure, recovers it again.*

XI. A Perfectly *elaſtick Body is ſuch a one, as recovers its firſt Figure with the ſame Forces whereby it was obliged to quit it.*

THEOR. XX.

If *two or more Bodies are carried by an equable Motion towards the ſame or contrary Parts, their common Center of Gravity before their mutual Concourſe will either be at reſt, or will move uniformly in a right Line.*

The firſt Caſe. Let the Bodies A and B tend towards contrary Parts, whoſe common Center of Gravity let be C.

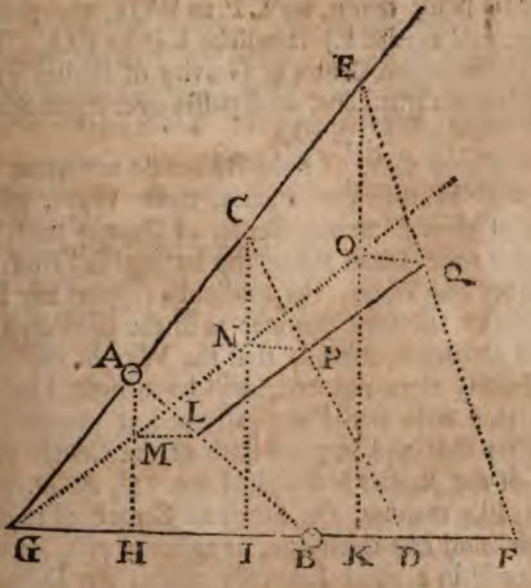


By reaſon of the equal Quantity of Motion in both Bodies, the Velocity of the Body A will be to the Velocity of the Body B, as the Body B to the Body A; that is, (from the Nature of the Center of Gravity) as A C to B C: whence, the Spaces paſſed over in the ſame Time being proportionable to the Velocities, whilſt the moving Body A runs through the Length A C, the Length B C will be run through by the moving Body B; ſo that the Bodies will meet in the Point C, and in that Point will be their Center of Gravity at the time of their Concourſe: but it was in the ſame Point before the Concourſe, ſo that it remained in the ſame place.

It may be ſhewn, after the ſame manner, that if the Bodies receded from the Point C with equal Motions, their Center of Gravity would have been at reſt.

The ſecond Caſe. If Bodies are carried in the ſame right Line towards the ſame Part, or with unequal Motions towards contrary Parts, their common Center of Gravity will be always found in the ſame right Line. For ſince the Bodies do uniformly and directly recede from one another, or approach one another, their Diſtance from one another will be uniformly increaſed or diminiſhed; and therefore the Bodies will uniformly recede from, or approach to any Point, dividing the aforeſaid Diſtance in a given Ratio. The Diſtance therefore of the Bodies from their common Center of Gravity will be uniformly increaſed or diminiſhed; which cannot be in the aforeſaid Caſes, unleſs that Center is either at reſt, (as in the firſt Caſe) or moves uniformly, as in the preſent Caſe.

The third Caſe. Let the Bodies A and B move in the right Lines A C, B D; and let the Spaces A C, C E, paſſed over by the Body A in equal Times, be equal; and the Spaces B D, D F, paſſed over by the Body B in the ſame Times, be alſo equal. Let the right Lines A C, B D, meet in G; and let it be as A C to B D ſo is A G to G H, and join A H; to which through C and E let C I, E K, be drawn parallel: it will be A C to H I as A G to G H, that is, as A C to B D; wherefore H I is = B D, and therefore H B = I D. In like manner, C E is to I K, as A G to GH, or AC to BD, that is, as CE to DF; wherefore IK is = DF, whence KF = ID = HB. Let L be the common Center of Gravity of the Bodies, when they are placed in the Points A and B; draw LM parallel to BD; the right Lines AB, AH, will be cut ſimilarly: join GM, and let it be produced; this will cut the Parallels to AH in the



Points N and O; *viz.* in the ſame Ratio as AH or AB is cut. Draw through N and O, parallel to BD, the right Lines NP, OQ; theſe will cut CD, EF, in the ſame Ratio as CI, EK, are cut, that is, in the ſame Ratio as AB is cut in L: but L is the common Center of Gravity of the Bodies when they are found in A and B; wherefore P will be their Center, when they are in the Points C and D, and Q when in E and F. Beſides, ML is to HB as AM to AH, or as CN to CI, or as NP to ID:

but HB and ID are equal; wherefore ML and NP will be equal: in like manner, NP and OQ will be equal. Since therefore the right Lines ML, NP, OQ, are equal and parallel, the right Line drawn through L, and parallel to MO, will paſs through the Points P and Q, and therefore the Center of Gravity will be always placed in the right Line LQ. Beſides, (by reaſon of the Parallels) AC is to CE as MN to NO, that is, as LP to PQ; wherefore (by reaſon AC is $=$ CE) it will be LP $=$ PQ. Therefore the common Center of Gravity of Bodies is always in the ſame right Line, and paſſes over equal Spaces in equal Times. *Q. E. D.*

The fourth Caſe. IF the Bodies do not move in any one, but in different Planes, their Ways, and the Way of their common Center of Gravity muſt be reduced to the ſame Plane, by letting fall from every Point of the Ways, Perpendiculars upon any Plane; and (after the ſame manner as in the preceding Caſe) it will be demonſtrated, that the Way of the Center of Gravity thus reduced, will be a right Line: and ſince this is in any Plane aſſumed at pleaſure, it is neceſſary that this way or Path of the Center of Gravity of the Bodies is a right Line. *Q. E. D.*

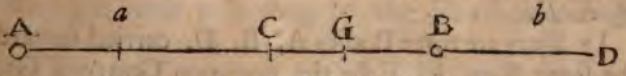
IN like manner, the common Center of theſe two Bodies, and any third one, is either at reſt, or moves uniformly in a right Line, by reaſon the Diſtance of the common Center of Gravity of the two Bodies, and the Center of the third, is divided by it in a given Ratio. And after the ſame manner the common Center of theſe three Bodies, and a fourth, either is at reſt, or moves in a right Line, by reaſon the Diſtance betwixt the common Center of the three, and the Center of the fourth Body, is divided by it always in the ſame Ratio; and ſo of any other number of Bodies. *Q. E. D.*

THEOR.

THEOR. XXI.

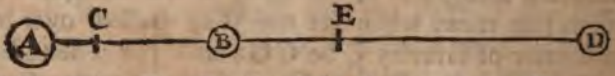
IF *two Bodies, whether equal or unequal, are carried towards the ſame Part, with Celerities whether equal or unequal, the Sum of the Motions in both Bodies will be equal to the Motion that would ariſe, if both the Bodies were carried with the Celerity of their common Center of Gravity.*

LET the two Bodies be A and B, whoſe common Center of Gravity let be C, and let both the Bodies be carried towards D; I ſay, the Sum of the Motions in both Bodies will be equal to the Motion that would be produced, if both Bodies were carried towards D with the Celerity of their Center of Gravity C. For let the Body A in any given Time deſcribe the Length A *a*, the Body B the Length B *b*, and in the mean while let the Way paſſed over by the Center of Gravity C be C G; and [by Theor. 6.] the Lengths A *a*, B *b*, C G, deſcribed in the ſame Time, will repreſent the Celerities of the Body A, the Body B, and the common Center of Gravity C reſpectively; but [by Corol. Theor. 3.] the Quantity of Motion in any Body is as a Rectangle contained under the Quantity of Matter and the Celerity, ſo that the Motion in the Body A will be as $A \times Aa$; and in the Body B as $B \times Bb$; and the ſum of the Motions will be as the Sums of theſe Rectangles, *viz.* as $A \times Aa + B \times Bb$. But [by the Def. of the Center of Gravity of Bodies] B C is to A C as A to B; and as A to B, ſo alſo [by the ſame Definition] is *b* G to *a* G: wherefore B C will be to A C as *b* G to *a* G; whence [by 19 *El.* 5.] B C is to A C, that is, A to B, as $BC - bG$ to $AC - aG$; that is, as C G

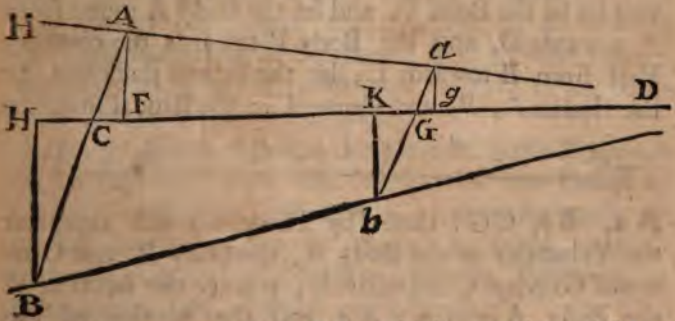


$-Bb$ to $Aa-CG$. So that [by 16 *El.* 6.] $A\times Aa-A\times CG$ will be equal to $B\times CG-B\times Bb$; and therefore $A\times Aa+B\times Bb$ will be equal to $A\times CG+B\times CG$: but the two Rectangles $A\times Aa$ and $B\times Bb$ are (as has been ſaid) as the Sums of the Motions in both the Bodies: and the two Rectangles under A and CG, and under B and CG, will be as the Sum of the Motions that would ariſe, if both the Bodies were carried with the Celerity CG of the Center of Gravity: whence the Sum of the Motions in both Bodies is equal to the Motion that would be produced, if both Bodies were carried with the Celerity of their common Center of Gravity. *Q. E. D.*

If there are three Bodies A, B, D, carried towards the ſame Part, let E be the common Center of Gravity of theſe three Bodies; the Sum of the Motions in the three Bodies will be equal to the Motion ariſing from the ſame Bodies carried with the Velocity of the Point E. For let C be the common Center of Gravity of any two Bodies A and B, the Motion in theſe two Bodies (by what has been demonſtrated before) will be equal to the Motion that would ariſe, if the Bodies coaleſced in one were carried with the Velocity of the Point C; but alſo the Sum of the Motions (*viz.* the Motion of the Bodies ſo coaleſcing, and the Motion of the third Body D) will be equal to the Motion which would happen, if the Body coaleſcing out of the two was moved together with the third Body D, with the Celerity of the Point E; whence the Theorem holds alſo in this Caſe.



The Demonſtration is the ſame, if the Bodies do not move in the ſame right Line, but in Parallels, or alſo in Lines any how inclined. But in this Caſe it muſt be obſerved, that the Celerity of the Bodies, wherewith they are carried towards the ſame Part with the Center of Gravity, is not to be reckoned by the Way which they really paſs over, but only by the Way wherein they move according to the Direction of the Center of Gravity. For example, if two Bodies A and B are carried in the right Lines A *a*, B *b*, and C G is the Line deſcribed by the common Center of Gravity, whilſt the Bodies paſs over the Lengths A *a*, B *b*, and from the Points A, *a*, B, *b*, let fall the Perpendiculars A F, *a g*, B H, *b* K upon the right Line C G. Now the Spaces which the Bodies paſs over according to the Direction of the

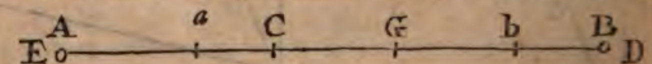


Point C, are not A *a*, B *b*, which are the abſolute Spaces deſcribed by them; but the Space according to which the Body A is moved towards the Part D, muſt be computed in the right Line F D, by the Length F *g*; for it moves ſo much, and no more, according to the Direction of the Point C. In like manner, the Space according to which the Body B is moved towards the Part D, is H K; and by that Space, its Progreſs in the right Line H D is to be reckoned: ſo that the Celerities of the Bodies wherewith they are carried towards the ſame Part, are as the right Lines F *g*, H K; beſides, A is to B as B C to A C, or (by reaſon of the equi-angular Triangles A C F, B C H) as H C to F C: whence the Demonſtration will proceed as in the firſt Caſe.

THEOR. XXII.

I F *two Bodies are carried towards contrary Parts, the Difference of the Motions towards the contrary Parts, or, which is the ſame thing, the Sum of the Motions towards the ſame Part, will be equal to the Motion that would be produced, if both the Bodies were carried towards the ſame Part with the Celerity of the common Center of Gravity.*

L E T the Bodies be A and B, whoſe Center of Gravity let be the Point C, and let the Body A move from A towards D, and the Body B towards the contrary Part from B towards E; let the Spaces deſcribed by the Bodies A, B, and Center C in the ſame Time, be



A *a*, B *b*, C G: theſe [by Theor. 6.] will repreſent the Velocities of the Body A, the Body B, and Center of Gravity C reſpectively; whence the Motion of the Body A is as $A \times Aa$, and the Motion of the Body B as $B \times Bb$, ſo that the Difference of the Motions will be $A \times Aa - B \times Bb$. Moreover, from the Nature of the Center of Gravity, B C is to A C as A to B; and as A to B, ſo will be *b* G to *a* G: wherefore it will be as B C to A C, ſo *b* G to *a* G. So that [by 19 *El.* 5.] B C will be to A C, that is, A to B as $BC - bG$ to $AC - aG$; that is, A will be to B as $Bb + CB$ to $Aa - CG$, wherefore [by 16 *El.* 6.] the Rectangle under A and $Aa - CG$ will be equal to the Rectangle under B, and $Bb + CG$; that is, $A \times Aa - A \times CG = B \times Bb + B \times CG$:

whence it will be $A \times Aa - B \times Bb = A \times CG + B \times CG$; but $A \times Aa - B \times Bb$ is (as was ſaid before) the Difference of the Motions towards the contrary Parts, or the Sum of the Motions towards the ſame, and $A \times CG + B \times CG$ is the Motion emerging, if both the Bodies ſhould be carried with the Velocity of their common Center of Gravity, whence the Propoſition is manifeſt.

Cor. 1. IF the Difference of the Motions towards the contrary Parts is equal to nothing, that is, if in both the Bodies the Quantities of Motion are equal, the common Center of Gravity in this Caſe will be at Reſt.

Cor. 2. IF there are ſeveral Bodies, either all carried towards the ſame Parts, or ſome towards the contrary Parts; the Sum of the Motions of all towards the ſame Parts will be the ſame, as if all were carried towards the ſame Part with the Velocity of the common Center of Gravity of them all.

Cor. 3. THE Motion therefore of many Bodies is to be reckoned from the Motion of the Center of Gravity, and their Syſtem will proceed forwards, or go backwards, aſcend or deſcend as much, as their common Center of Gravity proceeds forwards, or goes backwards, aſcends, or deſcends.

THEOR. XXIII.

IF *Bodies impinge on one another, or do act any way upon each other, the State of their common Center of Gravity of being at reſt, or moving uniformly in a right Line, will not be thereby changed.*

IF the Bodies impinge on one another, [by Theor. 19.] the Sum of the Motions towards the ſame Part will be the ſame before and after the Impulſe; but [by Theor. 21, and 22.] the Sum of the Motions before and after the Impulſe is the ſame, as if all the

Bodies were carried with the Velocity of the common Center of Gravity, and towards the ſame Part with it. Wherefore, ſince the ſame Bodies have the Sums of their Motions before and after the Impulſe equal to each other, and alſo equal to the Motion ariſing from all carried together with the Velocity of the common Center of Gravity: it is manifeſt, that the Velocity of the common Center of Gravity before and after the Impulſe will be the ſame. *Q. E. D.*

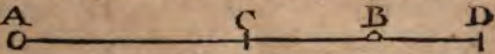
WE have hitherto delivered ſome general Laws, ſerving to determine the Motions of Bodies of all kinds; we ſhall now proceed to give ſome particular Rules relating to their meeting each other, *viz.* whereby each Body, after the Concourſe, and mutually ſtriking againſt one another, do continue their Motions, and towards what Parts they tend, and with what Velocity. But by reaſon of the different Structure of Bodies, *viz.* as ſome are endued with an elaſtick Force, whilſt others want that Force, the Rules of their Concourſe will be different according to the various Sorts of Bodies: and tho' perhaps there is no Body that is either perfectly hard, or perfectly ſoft, or perfectly elaſtick, (for haply all Bodies contain ſomething of all theſe in themſelves) yet it does not hinder, but that we may, by an Abſtraction of the Mind, ſeparate thoſe Qualities, and conſider a Body as only endued with one of theſe Qualities: and the Motions of Bodies will come the nearer to the Rules hereafter given, the more the Bodies themſelves partake of thoſe Qualities and Conditions.

WE here ſuppoſe the Bodies to be ſeparated from all others, that their Motions are neither hindered or promoted by ſuch as may lie round about them.

THEOR. XXIV.

If a hard or ſoft Body directly impinges or ſtrikes upon another hard or ſoft Body, whether the Body that receives the Stroke, is either at reſt, or moves ſlower towards the ſame Part with the other, or laſtly towards the contrary Parts, and the Motions are unequal, both Bodies after the Stroke will move together conjointly with the common Center of Gravity.

LET the Body A impinge on the Body B, which is either at Reſt, or moves ſlower towards the ſame Part, or is carried towards the contrary Parts with a leſs Motion; I ſay, that both the Bodies after the Impulſe, will move with the ſame Celerity, conjointly with the common Center of Gravity. For ſince the Body B is not hindered by the circumjacent Bodies, (by the ſecond Law) it will move by the Force impreſſed on it by the Body A, towards the Parts, wherein is the Direction of that Force; but it will alſo be moved conjointly with the Body A: For it cannot move ſlower, by reaſon the Body A follows immediately behind it; and it cannot move faſter, becauſe by Hypotheſis there is given no other impelling Cauſe of its Motion beſides the Body A; ſince we ſuppoſe all others, as an elaſtick Force, and an ambient Fluid, to have here no Influence: ſo that, after the Concourſe, both the Bodies and the common Center of Gravity will move conjointly together. *Q. E. D.*



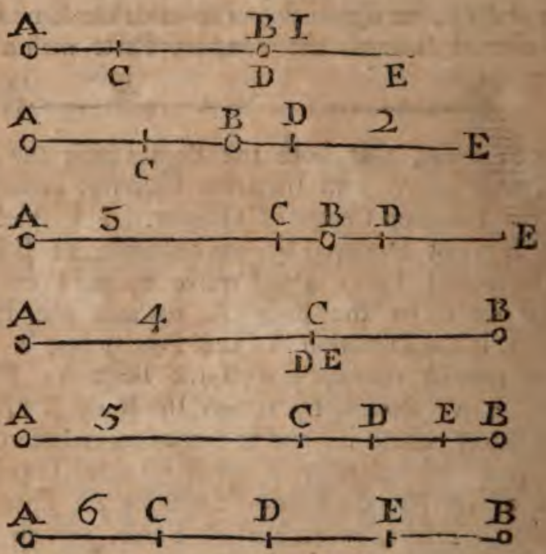
Cor. IF the Bodies are ſuppoſed to concur in D, ſince the Velocities of moving Bodies are as the Spaces deſcribed in the ſame Time, the Velocities of the Body A, the Body B, and the Center of Gravity

C, will be before the Concourſe, as the right Lines A D, B D, C D reſpectively; for theſe Lengths are paſſed over in the ſame Time.

PROB. II.

T o *determine the Motions of hard or ſoft Bodies after their direct Impact or Stroke.*

W e ſhall conſtruct all the Caſes of this Problem at once. Let therefore A and B be two Bodies, whoſe Center of Gravity let be C, and let us ſuppoſe the Bodies to meet in D, the Celerities of the Body A, of the Body B, and of the common Center of the Gravity C, before the Stroke, will be [by the preceding Corol.] as the right Lines A D, B D, and C D reſpectively. Let now D E be made equal to D C, this will repreſent the Velocity of the Bodies after



after their Concourſe; that is, the Velocity of the Body A before the Impulſe will be to its Velocity after the Impulſe as A D to D E: and the Velocity of the Body before the Stroke, will be to its Velocity after the Stroke, as B D to D E: for [by Theor. 19.] the Bodies A and B after the Impulſe move on together with the Center of Gravity; but [by Theor. 18.] the Celerity of the Center of Gravity will be the ſame both before and after the Impulſe, and always towards the ſame Part. Wherefore if C D repreſents its Velocity before the Impulſe, D E equal to C D will expreſs its Velocity after the Impulſe; ſo that D E will alſo repreſent the Celerity of the Bodies A and B, which after the Impulſe move together with the Center C. *Q. E. D.*

Cor. 1. If the Body B is at reſt, the Point D coincides with B, as in the firſt Figure; and becauſe B is to A, as A C to B C, or D E, by Compoſition it will be A + B to A, as A B or A D to D E; that is, the Velocity of the Body A before the Stroke is to its Velocity after the Stroke, as the Sum of the Bodies to the impinging Body A.

Example 1. If A is equal to the quieſcent Body B, it will be A + B to A as 2 to 1; ſo that the Velocity of the impinging Body will be double its Velocity after the Concourſe.

Example 2. If A is to B as 1 to 9, it will be A + B to A as 10 to 1; ſo that the Velocity after the Impulſe will be only the tenth part of the Velocity before the Impulſe.

Example 3. If B is a Body that exceeds A infinitely, the Velocity of the Body A, after the Impulſe, will be infinitely ſmall, that is, none at all; for in that Caſe, A in reſpect of A + B vaniſhes, and therefore the Velocity of the Body A after the Concourſe likewiſe vaniſhes: that is, if the Body A impinges on a firm and immoveable Obſtacle, after the Stroke it will be at reſt.

Example 4. If the Body B is equal to A, and moves ſlower in the ſame Direction, D E or C D will be

$$=\frac{AB}{2}+BD=\frac{AB+2\,BD}{2}=\frac{AD+BD}{2};$$

that is, the Velocity after the Impulſe will be half the Sum of the firſt Velocities.

Example 5. If the Bodies tend towards the ſame Parts with equal Motions, the Point D will coincide with C, as was demonſtrated in Theor. 20. and CD, DE, will be equal to nothing, that is, both the Bodies, will be at reſt, after their Concourſe.

Cor. 2. Hence the Law of the *Carteſians* is demonſtrated to be falſe, whereby they contend that there is always preſerved the ſame Quantity of Motion in the Univerſe: for Bodies which are not elaſtick, meeting each other from contrary Parts with the ſame Motions, mutually deſtroy each other's Motions.

Example 6. If equal Bodies tend towards contrary Parts with unequal Motions, it will be D E or C D

$$=CB-BD=\frac{AB}{2}-BD=\frac{AB-2\,BD}{2}=\frac{AD-BD}{2};$$

that is, the Velocity after the Impulſe will be half the Difference of the firſt Velocities.

All theſe things eaſily flow from the Conſtruction above; but ſince in Practice there is occaſion for a Calculation, a general Solution of this Problem may be thus diſcovered by Calculation.

Let the Velocity of the Body A be called C, and the Velocity of the Body B be c; and if the Bodies are moved according to the ſame Direction, the Sum of the Motions in both towards the ſame Part will be $AC+Bc$: but if they are moved towards contrary Parts, the Sum of their Motions towards the ſame Part will be $AC-Bc$; but [by Theor. 19.]

in all Bodies, the Sum of their Motions towards the ſame Part before and after the Impulſe is the ſame: wherefore the Motion of the Bodies after the Impulſe will be either $AC + Bc$ or $AC - Bc$, accordingly as the Bodies did tend towards the ſame or contrary Parts. There is therefore given the Momentum of the Bodies carried with the ſame Velocity, whence [by what has been ſaid in *Lect.* X.] their Velocity will be likewiſe known; namely, if their Momentum is divided by the Bodies themſelves, the Quotient will exhibit their Velocities, *viz.* $\frac{AC+Bc}{A+B}$ or $\frac{AC-Bc}{A+B}$; and if B is at reſt, that is, if c be put equal to nothing, the Velocity of the Bodies will be $\frac{AC}{A+B}$.

Cor. 3. SINCE the Velocity of the Body A before the Impulſe was as A D, and after the Impulſe its Velocity was as C D, the Velocity loſt will be A C, and therefore the Motion loſt by the Stroke $A \times AC$.

THEOR. XXV.

IF *a moving Body ſtrikes directly againſt another either in motion or at reſt; the Magnitude of the Stroke is proportionable to the Momentum that is loſt at the Concourſe by the more powerful Body, whichever of them it is.*

FOR if we ſuppoſe the more powerful Body, (whichever it is) or if they have equal Momenta, either of the Bodies, to be that which gives the Stroke, and the other that which receives it; the Magnitude of the Stroke will be equivalent to the Force impreſſed by the firſt on the other: wherefore that Force impreſſed departs from the percutient Body, [by the third Law] ſo that the Motion loſt in the percutient Body will be proportionable to

the

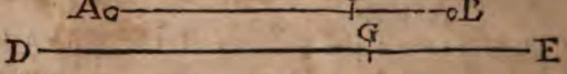
the Force impreſſed on the other Body, and conſequently to the Magnitude of the Stroke. *Q. E. D.*

Cor. WHERE the Momenta are equal that depart from the percutient Bodies, there the Magnitudes of the Strokes will be equal.

THEOR. XXVI.

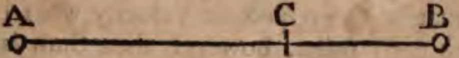
IF *a given Body directly impinges on another given Body at reſt, the Magnitude of the Stroke will be always proportionable to the Velocity of the impinging Body.*

LET the given Body A impinge on another given Body B that is at reſt, with a Velocity which may be expreſſed by A B. Afterwards let the ſame Body A impinge on the ſame quieſcent Body B, with another Velocity D E; that is, let A B be to D E as the former Velocity to the latter, and let the Diſtances of the Bodies be A B, D E: for whatever is the Diſtance at the beginning of Motion, it is the ſame thing in reſpect to the Magnitude of the Stroke; and let the common Center of Gravity in the firſt Situation be C, in the ſecond G. Since the Body A is moved with the Velocity A B, its Velocity after the Concourſe will be C B; and ſince its Motion before the Impulſe was A × A B, its Motion after the Impulſe will be A × C B; and the Motion loſt will be A × A C. After the ſame manner, if the Body is moved with the Velocity D E, the Motion loſt will be A × D G; and conſequently, the Magnitude of the Stroke with the Velocity A B will be to the Magnitude of the Stroke with the Velocity D E, as A × A C to A × D G, or as A C to D G: but becauſe it is A C to B C as B to A, it will be



AC to AC + BC, that is, AB, as B to A + B; and in like manner, it will be B to A + B as DG to DE. Wherefore it will be AC to AB, as DG to DE, whence by permutation it will be AC to DG as AB to DE; that is, the Magnitude of the Stroke with the Velocity AB, will be to the Magnitude of the Stroke with the Velocity DE, as the Velocity AB to the Velocity DE. *Q. E. D.*

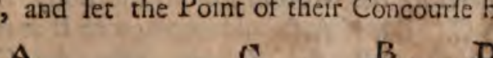
Cor. If the Body A ſhould ruſh on B, the Motion loſt would be as A × AC; but if B ſhould impinge on A with the ſame Velocity, the Motion loſt would be as B × BC; but becauſe A is to B as BC to AC, it will be A × AC = B × BC: ſo that the Quantity of Motion loſt by the Stroke, will be the ſame, whether B impinges on A with a given Celerity, or A with the ſame Velocity ſtrikes againſt B; and conſequently in both Caſes the Magnitude of the Stroke will be the ſame.



THEOR. XVII.

If *one Body ſhould impinge on another Body moving ſlower, but in the ſame right Line and towards the ſame Part, the Magnitude of the Stroke would be the ſame, as if the antecedent Body was at reſt, and the following Body was carried towards it with a Velocity equal to the Difference of their Velocities.*

Let the two Bodies carried towards the ſame Part be A and B, whoſe common Center of Gravity let be C, and let the Point of their Concourſe be D; from what has been delivered above, the Velocities of the Bodies before the Impulſe will be as the right



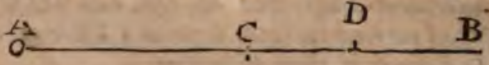
Lines A D, B D, and conſequently the Difference of their Velocities will be as A B: but the Velocity of both the Bodies after their Concourſe will be repreſented by C D, and conſequently the Motion loſt in the Body A will be A × A C. But if the Body A ſhould impinge with the Velocity A B on the quieſcent Body B, its Velocity after the Concourſe would be C B, and its Motion loſt A × A C: whence ſince in both Caſes the ſame Quantity of Motion is loſt in the percutient Body, the Magnitude of the Stroke will be alſo the ſame. *Q. E. D.*

Cor. IF the Difference of Velocities remains the ſame, that is, the reſpective Velocity whereby Bodies approach each other, however their Sum is increaſed or diminiſhed, the Magnitude of the Stroke will be always the ſame.

THEOR. XXVIII.

IF *two Bodies meet each other with contrary Motions, the Magnitude of the Stroke will be the ſame, as if one of them was at reſt, and the other impinged on it with the Sum of their Velocities.*

LET A and B be two Bodies carried towards contrary Parts, whoſe common Center of Gravity let be C, and let D be the Point where they concur; it's plain that the Velocities of the Bodies A and B are as the right Lines A D, B D; and conſequently the Sum of the Velocities will be repreſented by A B; but C D expreſſes their Velocity after the Impulſe, and therefore the Motion loſt in the Body A will be A × A C. But if A ſhould impinge on B at reſt, with the Velocity A B; the Velocity after the Concourſe would be as C B, and the Motion loſt as A × A C. Since therefore in both Caſes the ſame Quantity of Motion is loſt, the Magnitude of the Stroke will be the ſame. *Q. E. D.*



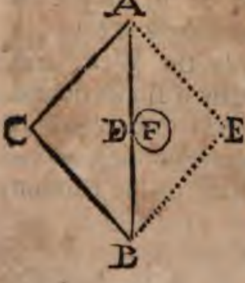
Cor. 1. IF therefore the Sum of the Velocities remains the ſame; that is, the reſpective Velocity of the Bodies A and B, wherewith they approach each other, whatever is the Difference of Velocities, or after what manner ſoever that Velocity is ſhared amongſt the meeting Bodies, the Magnitude of the Stroke will be always the ſame.

Cor. 2. THE Magnitude therefore of the Stroke in given Bodies is always proportionable to their reſpective Velocities.

Cor. 3. THE Motions of Bodies incloſed in a Space are the ſame amongſt one another, whether that Space is at reſt, or moves uniformly and directly on; for the Differences of Velocities, wherewith Bodies tend towards the ſame Part, and the Sums, wherewith they tend towards the contrary Parts, are the ſame, whether the Space, in which they are incloſed, is at reſt, or moves uniformly forwards: and therefore the Magnitudes of the Strokes being always proportionable to theſe, will be the ſame in both Caſes. Hence all the Motions within a Ship are performed in the ſame manner, whether it is at reſt, or is moved uniformly forwards. So alſo the Phenomena of Projectiles and Percuſſions do all happen juſt the ſame amongſt us placed upon the Earth, whether they are all together with the Earth in one common Motion, or there is no ſuch common Motion, and the Earth is at reſt; ſo that the Objections that are wont to be brought from the Inequality that ought to be in Projectiles not mov'd by the ſame Force, accordingly as they are made towards the Eaſt or towards the Weſt, and from the unequal Percuſſions made by a Bullet hurled by a Cannon, as the Exploſion is performed towards this or that part, and the like, prove nothing on either ſide, whether to eſtabliſh the Reſt or the Motion of the Earth.

LECTURE XIV.

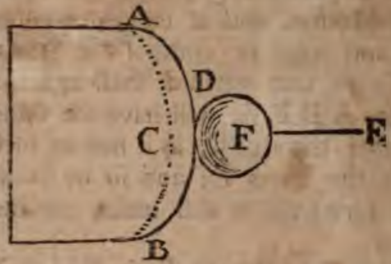
IF there were no ſuch thing as *Elaſticity*, the Laws, which we have propoſed in the preceding Lecture, relating to the Percuſſions of hard Bodies, would perfectly agree to all Bodies, and all Bodies after the Impulſe would tend conjointly towards the ſame Parts, towards which, before the Percuſſion, the more powerful Body did tend, that is, the Body whoſe Momentum was the greateſt; and with the Velocity that we determined in the before mentioned Laws. But ſince there are indeed very few Bodies, wherein there is not ſome degree of Elaſticity, (for ſoft Clay, Wax, and the like Bodies, contain in themſelves ſome Particles of Air, which gives them ſome Elaſtick Virtue) it happens by that Elaſtick Force, that Bodies after their Impulſe do not move conjointly together, but fly from one another, and are moved with a different Velocity, ſometimes towards the ſame, and ſometimes towards the contrary Parts. But in order to underſtand the Modus and Cauſe of this Rebounding in Bodies, we will illuſtrate the thing by an Example.



LET AB be a String extended over a Plane, but at ſome Diſtance from it, whoſe Extremities AB let be firmly fixed, and the String itſelf very much ſtretched: Now if the String be drawn by its middle Point D, its Extremities remaining immoveable, into the Situation ACB, ſo that its Point D ſhall be in C, and then be let go, the String will not remain in the Situation A C B, but with a great Force will endeavour to reſtore itſelf to its former Situation: and ſince by the continual Action of the Elaſtick Force a ſufficiently ſwift Motion is raiſed in the String, it will happen that when its Motion perſeveres towards the ſame Part, till the Elaſtick or Reſtoring Force, conſtantly reſiſting this farther Motion, and at length equalling it, ſhall deſtroy it, and urge or compel the String towards the Parts C; ſo that when it ſhall again arrive into the Situation A D B, it will have the ſame Force of moving farther towards C, as it had at firſt of tending towards the Parts E: and ſo by moving backwards and forwards, it will make continual Vibrations.

LET us now ſuppoſe the Body F to ſtrike againſt the String A B, the String will be then compelled to quit its Situation through the Force exerted on it by the Body F; and its Point D, on which the Body F gave the Stroke, will be moved, together with F, towards C: which Motion will continue even till the reſtoring Power of the String, which is contrary to the Motion of F, ſhall be equal to that Motion; which as ſoon as it is, all the Motion towards C will be deſtroyed. But this elaſtick Force acting farther, will bring back the String, which therefore will compel the Body F, and move it along with it with the ſame Velocity. But (by reaſon we ſuppoſe the Tenſion of the String to be very great) the String will reſtore itſelf with the ſame Force, wherewith it was firſt inflected: now the Force wherewith it was inflected, was equivalent to the Momentum of the impinging Body, (for that was all ſpent in bending the String) ſo that the String by acting on the Body F with ſuch a Force, will reſtore to it the ſame Quantity of Motion, as was ſpent in bending it; and therefore the Body F will be ſent back with the ſame Velocity that it came forwards with, and ſo a Reflection will be occaſioned.

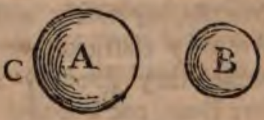
LET us now put, inſtead of a String, any elaſtick Body A B, which at firſt we ſhall ſuppoſe fixed and immoveable, and let its Superficies A D B be preſſed inwards by the Force of the Body F ſtriking againſt it. As ſoon as the compreſſing Force, that is, the Motion of the Body F ſhall ceaſe, the ſpringing Body by its innate Force will reſtore itſelf to its former Figure, and by that Force will compel the Body F towards E; and if both the Bodies are perfectly elaſtick, the reſtoring Force of the Spring will be equivalent to the Force that compreſſes it, that is, to the Momentum of the Body F: and therefore it acting on the Body F with this Force, it will compel that Body to return back with the ſame Velocity which it had at firſt. But if the Body A D B C is not fixed, but in ſuch a ſtate as its Motion is not hindered by any other Body, then the elaſtick Force will equally exert itſelf on both the Bodies, and it will produce equal Changes of Motions: for if the Body A D B compels the Body F towards the Part E, that again will be equally compelled by the Body F towards the contrary Part; and therefore the Bodies will rebound from each other. And ſo we have demonſtrated after what manner it happens, that Bodies after the Impulſe



pulſe do not either reſt, or move conjointly together, but by rebounding from one another, they move with different Velocities, ſometimes towards the contrary Parts, and ſometimes towards the ſame.

THE *Carteſians*, who were ignorant of the Force of Elaſticity to reflect Bodies, having given a quite different Cauſe of Reflection: for they ſay, that Motion is not contrary to Motion, but its Direction to its Direction; and therefore one Body ſtriking againſt another, is reflected, becauſe the Motion of the Body that gives the Stroke cannot be deſtroyed, ſince according to them nothing can be contrary to Motion: but ſince the Direction of one oppoſes the Direction of another, they will have it, that the ſtriking Body after the Impulſe muſt be reflected towards the contrary Parts, the Quantity of Motion remaining the ſame in the Body that gives the Stroke, and in that which received it.

BUT it is eaſy to ſhew, that this Notion is neither agreeable to Reaſon or Experience: for ſince the Momentum or Quantity of Motion conſtitutes that Force or Energy wherewith the moving Body tends according to its Direction, if two Bodies directly meet each other, the Forces impreſſed according to their contrary Courſes, will be contrary: ſo that if they are equal, they will mutually deſtroy each other; if unequal, the Motion that is of the leſs efficacy, will be deſtroyed. Beſides, one Body ſtriking againſt a greater Body at reſt, or moving ſlower towards the ſame Parts, is reflected; but this cannot be done only by reaſon the Direction is contrary to the Direction: for if the Body B ſtrikes againſt another greater Body A, that is either at reſt, or moves ſlower towards the ſame Parts; ſince all the Force that is found in both the



the Bodies, tends towards C, that Force can never direct the Motion in both the Bodies towards the contrary Parts. For [by the ſecond Law] all Motion is performed according to the right Line, wherein the Force is impreſſed; but [by Hypotheſis] all the Force is impreſſed according to the right Line B C, from B towards C: wherefore if the Reflection of Motion was only made by the Force that is innate in Bodies, without a new Force, there would be cauſed a Motion in a Courſe that is contrary to that wherein the Force is impreſſed; which cannot be. Therefore that Reflection does not ariſe from the Force firſt impreſſed, but from an Elaſtick Force, with which both the Bodies are endued, and which equally acting both ways, compels the Bodies to rebound from one another.

BESIDES, if Motion was not contrary to Motion, it would be much eaſier to direct a Body that is once in motion, towards the contrary Parts, than to ſtop it quite: for in the firſt Caſe, the Motion of the Body reflecting on one's Hand is not received, but only turned towards the contrary Parts; but in the latter Caſe, all that Motion is employed on the reſiſting Body, which yet is contrary to manifeſt Experience. Laſtly, if nothing was contrary to Motion, where-ever any Body hit againſt any Obſtacle, there will always be a Reflection, which is nevertheleſs contrary to Obſervation: for Lead, Clay, Wax, and other Bodies that have ſcarce any Elaſticity in them, if they fall on the Pavement, are not reflected; yet when Balls made of Wool or Feathers, Ivory, Marble, or Glaſs, and other the like Bodies, which are very elaſtick, are thrown againſt the ſame Pavement, they ſtrongly rebound: That Reflection therefore does not proceed from the Motion that is common to all Bodies, but from an Elaſticity which is only peculiar to reflecting Bodies. Which was to be ſhewn.

BUT perhaps the *Carteſians* may aſk, how we come to know that Balls of Ivory, Glaſs, and Marble, and other reflecting Bodies, which ſeem to be exceeding hard, are endued with an Elaſticity: I anſwer, That they are elaſtick, may be hence concluded, That when they are ſtruck, they yield a Sound, which ariſes from the Vibrations of the Body that receives the Stroke, after the ſame manner as a ſtretched String, by its Vibrations cauſes Undulations in the Air; and therefore it is not in the leaſt to be doubted, but that theſe Bodies are endued with ſome Spring. And this Argument indeed makes it probable that theſe Bodies have an elaſtick Force, but there is another Argument, whereby this matter may be demonſtratively proved.

FOR let there be two Balls either of Ivory or Glaſs, and if their Figures were perfectly ſpherical, they would touch one another in one only and that an indiviſible Point; but this cannot be done by any human Art: yet they may be made ſo nearly ſpherical, that they ſhall touch one another in a phyſical Point, that is, in the leaſt viſible part. Now if the Superficies of one of the Balls is ſtained with Ink (or with any Colour that is eaſily wiped off) and another Body impinges on it at reſt, it will appear from the Experiment, that not only a phyſical Point of the Ball that ſtrikes againſt the other, will, after the Impulſe be ſtained with the Colour of that other Ball, but a ſufficiently large Part thereof; and this could not be, unleſs their Superficies were changed by the Force of the Stroke: but after the Reflection, we find that both the Balls have recovered their firſt Figure. Wherefore the Balls have an elaſtick Force, whereby they are able to recover their former Figure that was alter'd by the Stroke. *Q. E. D.*

NOW follow the Rules of Motion obſerved by Elaſtick Bodies.

THEOR. XXIX.

IF *two perfectly Elaſtick Bodies impinge on one another, their relative Velocity will be the ſame before and after the Impulſe; that is, the perfectly Elaſtick Bodies will recede from each other after the Stroke, with the ſame Celerity, wherewith they firſt approached one another.*

FOR [by Cor. Theor. 27.] the compreſſing Force, on the Magnitude of the Stroke in given Bodies, ariſes from the relative Velocity of the Bodies, and is proportionable to it; and [by Def. 11.] perfectly Elaſtick Bodies reſtore themſelves to their former Figure by the ſame Force, whereby they were compreſſed: that is, the reſtoring Force is equal to the compreſſing Force, and therefore is equivalent to the Force wherewith the Bodies approach each other before the Impulſe. But Bodies are compelled to ſeparate from one another by this reſtoring Force, whence this Force acting on the ſame Bodies, will produce a relative Velocity equal to that which they had at firſt, or it will make the Bodies to recede from one another with the ſame Velocity wherewith they at firſt approached each other. *Q. E. D.*

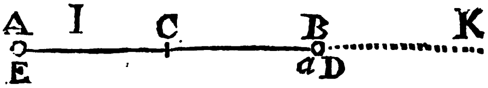
Cor. THEREFORE in equal Times taken before and after the Impulſe, the Diſtance of the Bodies from one another will be equal; and conſequently in the ſame Times, the Diſtances of the Bodies from the common Center of Gravity will be alſo equal.

FROM this Corollary, the Rules of Motion in the Congreſs of perfectly Elaſtick Bodies may be eaſily diſcovered; which we ſhall therefore do in the following Problem.

PROBL. III.

To determine the Rules of Congreſs in Bodies that are perfectly Elaſtick, and which do impinge directly on one another.

WE ſhall give at once the Conſtruction of all the Caſes of this Problem. Let A and B be two perfectly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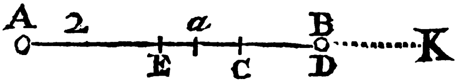
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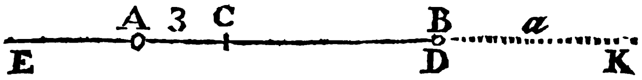
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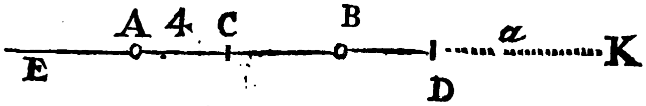
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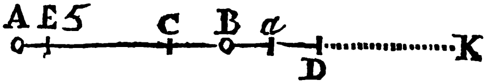
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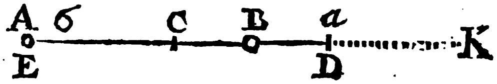
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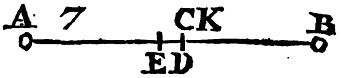
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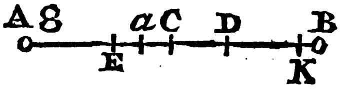
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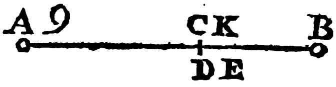
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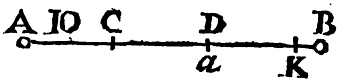
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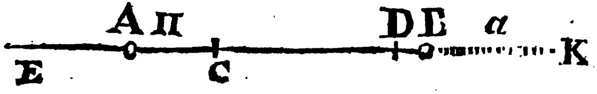
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fectly Elaſtick Bodies, whoſe common Center of Gravity let be C, and let the Bodies meet in D; make C E equal to C D: I ſay, that after the Concourſe the right Line E A will expreſs the Velocity of the Body A from E towards A, and the right Line E B will expreſs the Velocity of the moving Body B from E towards B.

Demonſtration. SINCE [by Theor. 23.] the common Center of Gravity of the Bodies before and after the Impulſe, proceeds always uniformly with the ſame Velocity, in the Time equal to that wherein the Length A D is paſſed over by the Body A, or the Length C D by the Center of Gravity C, after the Impulſe the Length D K equal to D C, will be paſſed over by the ſame Center C; make K *a* equal to C A: and ſince [by the Corol. of the preceding Theor.] in equal Times taken before and after the Impulſe, the Diſtances of the Bodies from the common Center of Gravity are always equal, in the ſame Point of Time, as the common Center of Gravity is in K, the Body A will be found in *a*, ſo that after the Impulſe its Motion will be from D towards *a*; and its Velocity will be as the right Line D *a*, which is paſſed over by it in that Time. But by reaſon C E is equal to the right Line C D or K D, and C A equal to K *a*, the Difference of the right Lines C E, C A, will be equal to the Difference of the right Lines K D, K *a*, that is, E A will be equal to D *a*; but the right Line D *a* denotes the Velocity of the Body A after the Impulſe: wherefore its Velocity will be alſo denoted by the right Line E A. Beſides, ſince the relative Velocity of the Bodies before and after the Impulſe is the ſame, and the right Line E A denotes the Velocity of the moving Body A, the Velocity of the moving Body B, after the Impulſe muſt be neceſſarily denoted by the right Line E B; namely, from E towards B. *Q. E. D.*

Cor. 1. IF the Body B is at reſt, the Point D will coincide with B, as in the firſt three Figures: and becauſe B is to A as A C to C B, it will be by Compoſition B + A to A as A B to C B: whence by doubling the Conſequents, it will be B + A to 2 A, as A B to 2 C B or E B; that is, as the Aggregate of the Bodies to twice the impinging Body, ſo is the Celerity of the impinging Body before the Impulſe, to the Celerity of the Body that is firſt at reſt after the Contact.

Cor. 2. AND therefore if A and B are equal, it will be $A + B = 2A$; whence E B, the Celerity of the Body B, after the Contact will be equal to A B the Celerity of the Body A before the Contact, and conſequently the Point E coinciding with the Point A, A E the Velocity of the moving Body A after the Impulſe will be equal to nothing: which may be alſo thus eaſily ſhewn: By reaſon of the Bodies A and B being equal, it will be $AC = CB = CD = CE$, wherefore the Point E coincides with A, and conſequently the moving Body A, after the Impulſe, will be at reſt, and the Body B after the Impulſe will move with the Celerity E B or A B. If therefore an elaſtick Body impinges on another equal to it and at reſt, after the Contact the impinging Body will be at reſt, and that which was at reſt will be moved with the Celerity of the firſt Body.

Cor. 3. IF the equal Bodies A and B are carried towards the ſame Part (as in Fig. 4.) after the Contact they will be alſo carried towards the ſame Part with interchanged Velocities. For by reaſon C E is = C D, and $AC = CB$, it will be $CE - AC$, that is, $EA = CD - CB$ or B D; and therefore the Velocity of the Body A after the Impulſe will be equal to the Velocity of the Body B before the Impulſe. Beſides, becauſe E A is = B D, it will be $EB = AD$; and conſequently the Velocity of the Body B after the Contact will be equal to the Velocity that the Body A had before the Concourſe.

Cor. 4. If the equal Bodies A and B are carried towards the contrary Parts, (as in Fig. 8.) they will after the Impulſe recede towards the contrary Parts, with interchanged Velocities. For by reaſon AC is $= CB$ and $CE = CD$, it will be $AC - CE$, that is, $AE = CB - CD$ or BD; and therefore the Velocity of the Body A, after the Impulſe, will be equal to the Velocity the Body B had before the Impulſe. Beſides, by reaſon EA is $= BD$, it will be $AD = EB$; but AD was the Velocity of the Body A before the Concourſe, and EB is the Velocity of the Body B after the Concourſe, whence the Corollary is evident.

But ſince Calculation is always uſed in Practice, we ſhall here deliver a Method, whereby the Celerities of Elaſtick Bodies after the Impulſe may be inveſtigated, and brought to Numbers; and it would indeed be eaſy, after the manner of the foregoing Corollaries, to reduce to Numbers all the particular Caſes from the general Conſtruction; but a general Calculation may be thus more readily diſcovered.

Let us in the firſt place ſuppoſe the Bodies A and B to be moved towards the ſame Part; and let C be the Velocity of the following Body A, but c the Velocity of the preceding Body B: whence the relative Velocity of the Bodies will be $C - c$, and the Sum of the Motions towards the ſame Part $AC + Bc$. Let the Velocity of the Body A after the Impulſe towards the ſame Part, as before, be called x; and becauſe the relative Velocity of the Bodies both before and after the Concourſe is the ſame, the Velocity of the Body B will be $x + C - c$: for the relative Velocity of the Bodies is equal to the Exceſs of the Velocity, whereby the Velocity of the ſwifter Body exceeds that of the

the ſlower, ſo that the Exceſs ought to be $C - c$; but ſince the Velocity of the Body A is x, its Motion towards the Part D will be $A x$; and ſince the Velocity of the Body B is $x + C - c$, its Motion towards the ſame Part will be $B x + B C - B c$; and the Sum of theſe Motions will be equal to the Sum of the former Motions, that is, it will be $A x + B x + B C - B c = A C + B c$: whence by reducing this Equation, it will be $A x + B x = A C - B C + 2 B c$; and $x = \frac{A C - B C + 2 B c}{A + B}$ = the Velocity of the Body A. Moreover, the Velocity of the Body B is $= x + C - c = \frac{A C - B C + 2 B c}{A + B} + C - c = \frac{A C - B C + 2 B c + A C + B C - A c - B c}{A + B} = \frac{2 A C - A c + B c}{A + B}$.

If B C is greater than $A C + 2 B c$, x or $\frac{A C - B C + 2 B c}{A + B}$ will be a negative Quantity; ſo that the Velocity of the Body A in that caſe will be towards the contrary part, and its Motion towards D will be negative. If the Body B is at reſt, that is, if $c = 0$, the Velocity of the Body A after the Impulſe will be $+ \frac{A C - B C}{A + B}$ forwards or backwards, as the Sign + or — ſhall prevail.

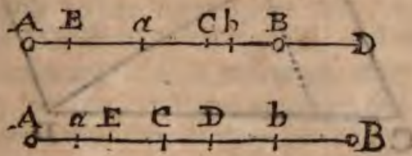
If the Bodies A and B carried towards the contrary Parts with the Celerities C and c, do directly impinge on each other, their Motion towards the ſame Part will be $A C - B c$, and the relative Velocities of the Bodies will be $C + c$. Now let x be the Velocity of the Body A after the Impulſe; its Motion towards the ſame Part as before, will be $A x$, and the Velocity of the Body B will be $x + C + c$, (for the relative Velocity of Bodies is not altered by the Stroke) and the Motion in the Body B towards D will be $Bx+BC+Bc$; whence the Sum of the Motions towards the ſame Part will be $Ax+Bx+BC+Bc$, which [by Theor. 14.] will be $=AC-Bc$: ſo that it will be $Ax+Bx=AC-BC-2Bc$, and $x=\frac{AC-BC-2Bc}{A+B}$, and the Velocity of the Body B will be $\frac{AC-BC-2Bc}{A+C}+C+c$ $=\frac{AC-BC-2Bc+AC+Ac+BC+Bc}{A+B}=\frac{2AC+Ac-Bc}{A+B}$.

If $BC+2Bc$ is greater than AC, the Motion of the Body A will be backwards; namely, towards the conttary Part: in which caſe x or $\frac{AC-BC-2BC}{A+B}$ will be a negative Quantity.

The firſt, as I know of, who gave the true Laws of Motion in hard Bodies, was Dr. *John Wallis*, the famous *Savilian* Profeſſor of Geometry in this Univerſity, which he did in the *Philoſophical Tranſactions* N° 43. where he alſo ſhew'd the true Cauſe of Reflections in other Bodies, and proved that they proceeded from their Elaſticity. Not long after, the famous Sir *Chriſtopher Wren*, and Monſ. *Huygens*, imparted to our *Royal Society* the Laws that are obſerved by perfectly Elaſtick Bodies, and gave exactly the ſame Conſtruction, tho' each was ignorant of what the other had done. But ſince they publiſhed in the *Philoſophical Tranſactions*, the Conſtructions and Laws of Motion without any Demonſtration; we have thought fit to take thence their very elegant Conſtruction, and to demonſtrate it.

After a not unlike Method is conſtructed the Problem in ſuch Bodies as are indeed Elaſtick, but which do not reſtore themſelves with a Force equal to that whereby they were compreſſed. For let there be any two Bodies A and B, whoſe common

Center of Gravity let be C; let A C, B C, be ſo cut in *a* and *b*, that A C is to *a* C and B C to *b* C, as the



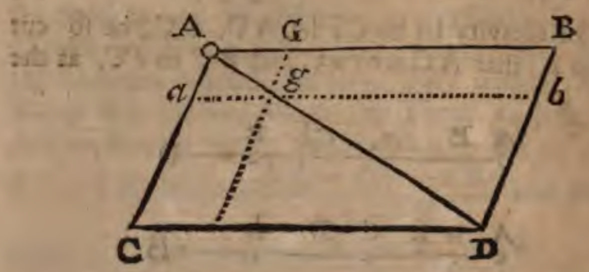
Force compreſſing the Elaſtick Body to the Force whereby it reſtores itſelf: and let C E be equal to C D, E *a* will be the Velocity of the Body A after the Impulſe from E towards *a*, and E *b* will be the Velocity of the Body B from E towards D.

But if the reſtoring Force is equal to the compreſſing Force, the Point *a* will coincide with A, and the Conſtruction returns to the former. The Demonſtration is eaſy to any one that underſtands the preceding, nor is it neceſſary to add it here.

THEOR. XXX.

If *a Body* A *is moved uniformly in the right Line* A B, *and in the mean while that right Line* A B *is carried always parallel to itſelf, as alſo with an equable Motion according to a Direction that is parallel to* A C; *and let the Velocity of the Body* A *be to the Velocity of the Line* A B *as* A B *to* A C, *and let be compleated the Parallelogram* A B D C, *whoſe Diagonal let be* A D: *this will be the true Line deſcribed by the Body* A *in its Motion.*

When the Line A B ſhall come to the Situation *a b*, let *g* be the Place of the Body A; and becauſe [by Theor. 6.] the Spaces deſcribed in the ſame Time are as the Velocities, *a g* the Length paſſed over by the moving Body A, will be to A *a* the Length paſſed over by the Line A B, as the Velocity of A to the Velocity of the right Line A B, that is [by Hypotheſis]



ſis] as A B to A C: whence the Parallelogram *a* G will be ſimilar to the Parallelogram C B, and conſequently (by 24 *El.* 6.] the Point *g* will be placed in the Diagonal A D; that is, the Body A will be always found in the right Line A D: ſo that this Line will be paſſed over by it. *Q. E. D.*

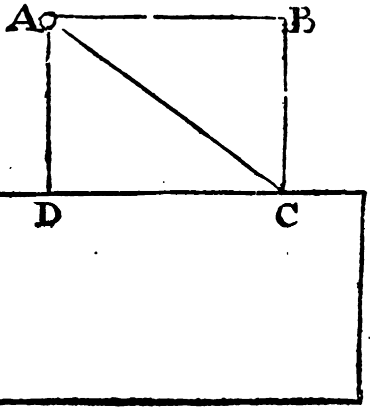
Cor. 1. THE Line A D will be deſcribed by the moving Body A in the ſame Time, wherein, without the Motion according to A C, it would paſs over the Line A B; or wherein, without the Motion according to A B, it would deſcribe the right Line A C.

Cor. 2. SINCE the moving Body is therefore carried in the right Line A D, becauſe beſide its own proper Motion, it likewiſe partakes of the Motion of its Place, or of the right Line A B, and its Motion is compounded of both theſe; if any moveable Body has two Motions impreſſed on it at the ſame time, according to the Directions A B, A C; and thoſe Motions, or the Forces whereby they are produced, are as the right Lines A B, A C: A D will be the Line deſcribed by the moving Body, that receives the Motions impreſſed by theſe two Forces, and its Force, whereby it is carried in the right Line A D, will be to the Forces according to A B, A C, as the Diagonal A D to the Sides of the Parallelogram A B, A C.

Cor. 3. HENCE converſly, if the moving Body with a Force as A D paſſes over the right Line A D, the Motion will be the ſame, and according to the ſame Direction, as if at the beginning of the Motion it had been impelled by two Forces at once, each proportionable to the right Lines A B, A C, according to the Directions from A to B, and from A to C. And hence any Motion whatever, tho' ſimple in it ſelf, may be conſider'd as compounded of more Motions; and any Forces may be reſolved into others, that are more in number, and act according to different Directions.

THEOR. XXXI.

IF *the Body* A *impinges obliquely on a firm Obſtacle* D C, *the Energy of the Percuſſion, or Magnitude of the oblique Stroke, will be to the Magnitude of the Stroke that the ſame Body would have produced, if it had impinged perpendicularly with the ſame Celerity, as the Sine of the Angle of Incidence* A C D *to the Radius.*



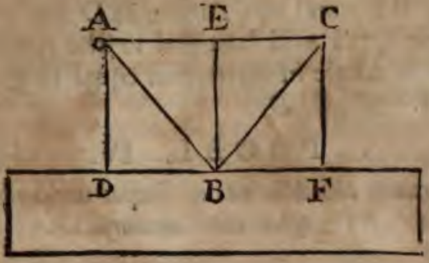
FROM A let fall on the Obſtacle the Perpendicular A D, if the Superficies of the Obſtacle is a Plane; or if a Curve, let the Perpendicular be let fall on the

Plane touching the Obſtacle in the Point of Incidence C, and compleat the Rectangle D B. Now [by Cor. 3. of the preceding Theor.] the Motion of the Body A being as A C in the right Line A C, will be equivalent to two Motions impreſſed at the ſame time according to the Directions A B, A D, which are to the Motions in A C as the right Lines A B, A D, to A C: but the Obſtacle does no ways reſiſt the Motion in the right Line A B; for ſince A B is parallel to D C, the Body moved in the right Line A B will never impinge on the Obſtacle D C. The Force therefore wherewith it impinges on the Obſtacle, is as the right Line A D: the Force therefore of the Body A in the right Line A C, is to the Force wherewith it impinges on the Obſtacle, as A C to A D. But if it had impinged perpendicularly on the ſame with a Force as A C, the Magnitude of the Stroke would have been repreſented by A C, for the whole Motion would have been deſtroyed by the Obſtacle: wherefore the Magnitude of the oblique Stroke will be to the Magnitude of the perpendicular Stroke as A D to A C; that is, A C being made the Radius, as the Sine of the Angle of Incidence to the Radius.

THEOR. XXXII.

IF *a perfectly Elaſtick Body impinges obliquely on a firm Obſtacle, it will be ſo reflected from it, that the Angle of Reflection ſhall be equal to the Angle of Incidence.*

LET the perfectly Elaſtick Body A fall obliquely on the firm Obſtacle according to the Line A B; I ſay, that will with the ſame Celerity be ſo reflected in the right Line B C, that the Angle of Reflection C B F ſhall be equal to the Angle of Incidence A B D. Let the right Line A B expreſs the Motion of the Body A in the Direction A B: [by Corol. 3. Theor.



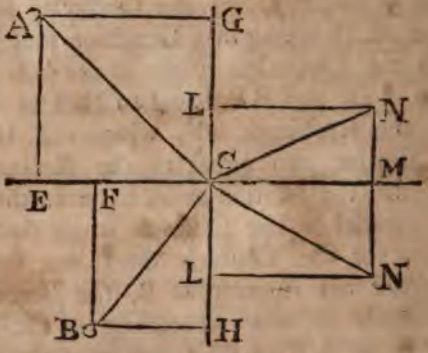
Theor. 30.] let this Motion be reſolved into two others, according to the Directions, A E, A D, to which the Motion in A B is as A B to A E, A D: but ſince A E is parallel to the Superficies of the Obſtacle, and A D perpendicular to it, or at leaſt to the Plane touching the Obſtacle in B; that Force wherewith it impinges on the Obſtacle, is only that which is as A D acting in a Direction that is perpendicular to the Obſtacle. Now make B E equal and parallel to A D, and B F equal to D B or A E, and compleat the Rectangle E F, which will be in every thing ſimilar and equal to the Rectangle D E. Since therefore the Motion as A E, according to the Direction parallel to the Obſtacle is not deſtroyed by the Stroke, for the Obſtacle does not oppoſe this Motion; after the Impulſe at B, the Force remaining in the Body to move according to the Direction B F, will be as A E or B F: but from the Nature of Elaſticity, a Body impinging on an Obſtacle with a Force as E B according to the Direction E B, is reflected with the ſame Force according to the ſame Direction; the Motion therefore of the Body at the Point of Incidence B is compounded of the Motion as B F according to the Direction B F, and the Motion as B E according to the Direction B F. Wherefore [by Corol. 2. Theor. 30.] the Body will be moved in the right Line B C with a Force as B C: but by

reaſon A D, C F, are equal and parallel, and by reaſon D B, B F, and the Angles at D and F are equal, the Angle C B F will be equal to the Angle A B D, that is, the Angle of Reflection equal to the Angle of Incidence. *Q. E. D.*

PROBL. IV.

T o *determine the Motions of Bodies impinging obliquely after the Concourſe.*

Let any Bodies A and B be moved in the Lines A C, B C, inclined to one another, whoſe Lengths let reſpectively expreſs the Velocities of the Bodies A, B: let E F C repreſent the Plane whereby the Bodies are touch'd in the Point of Concourſe; on which from A and B let fall the Perpendiculars A E, B F, which may expreſs the Velocities wherewith the Bodies approach each other. Compleat the Rectangles E G, F H. [By Cor. 3. Theor. 30.] the Motion of the Body A is reſolved into two others, according to the Directions A G, A E; to which the Motion in A C is as A C to A G, A E, reſpectively: in like manner, the Motion of the Body B is reſolved into two other, according to the Directions B F, B H;



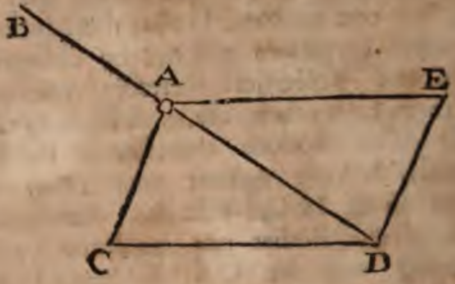
BH; to which the Motion in BC is as BC to BF, BH, reſpectively. But ſince AG, BH, are parallel, by the Velocities wherewith the Bodies are moved according to theſe Directions, they will not impinge on one another, ſo that the Motion according to theſe Directions will not be changed by the Impulſe; the Velocities therefore wherewith the Bodies meet each other, are as AE or GC and BF or HC. The Motions therefore of the Bodies A, B, directly ſtriking againſt each other with the Velocities GC, HC, will be determined, [by Probl. 2. if the Bodies are hard, or by Probl. 3. if elaſtick;] and let CL be the Velocity of the Body A from C towards L after the Impulſe, which ariſes from the Velocities GC, HC. And ſince, as has been ſhewn, there remains in the Body a Force of moving according to the Direction parallel to AG, with a Velocity as AG, make CM equal to AG, and compleat the Rectangle LM: the Body A, after the Impulſe, will move in its Diagonal CN with a Velocity as CN, as appears by Corol. 2. Theor. 30. and after the ſame manner will be determined the Motion of the Body B after the Impulſe. *Q. E. F.*

THEOR. XXXIII.

IF *a moveable Body* A *is drawn by three Powers by the means of three Strings, or by any other manner whatever is ſollicited according to the Directions* AB, AE, AC, *ſo that theſe three Powers are equivalent to each other, that is, if any two of them deſtroy the Effect of the other, and the Body is moved by none of them; theſe Powers will have amongſt themſelves the ſame Ratio, as have the right Lines parallel to their Directions, and terminate by their mutual Concourſe.*

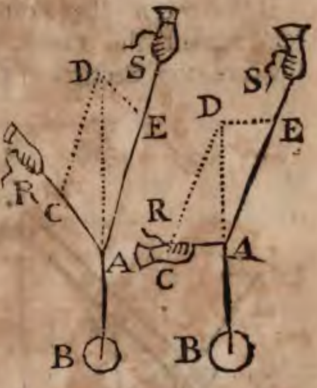
LET AD expreſs the Power or Force wherewith the moveable Body A is ſollicited from A towards B;

the Force equivalent or equal to this, and ſolliciting the Body A the contrary way from A towards D, will be alſo expreſſed by A D: but [by Cor. 3. Theor. 30.] the Force impelling the Body from A towards D, will be equivalent to two Forces acting according to the Directions A C, A E, to which the former Force acting from A towards D, is as A D to A C, A E, or to A C, C D, reſpectively; and inverſely the Forces acting according to the right Lines A C, A E, and equivalent together to the Force ſolliciting the Body from A towards D, ought to be to the ſame Force according to A D as A C and A E or C D to A D: wherefore likewiſe the Forces acting according to the right Lines A C, A E, and equivalent to the Force wherewith the Body is ſollicited from A towards P, and deſtroying its Effect, ought to be to the ſame, as A C, C D, to A D; that is, if the ſame moveable Body is ſollicited by three Powers equivalent to each other, according to the Directions A B, A C, A E, theſe three Powers will be as the right Lines A D, A C, A E, reſpectively. *Q. E. D.*



Cor. 1. SINCE in every Triangle the Sides are as the Sines of the oppoſite Angles, A C will be to C D as the Sine of the Angle A D C or D A E to the Sine of the Angle D A C: whence any two Powers will be amongſt themſelves reciprocally as the Sines of the Angles, which the Lines of their Directions contain with the Line of Direction of the third Power. Beſides, A D is to A C as the Sine of the Angle C or A E D to the Sine of the Angle C D A or D A E; and in like manner, the Power acting according to A B, is to the Power according to A E as the Sine of the Angle A E D to the Sine of the Angle A D E or C A D.

Cor. 2. If two Powers R, S, ſuſtain a Weight B, by the means of Strings drawing in the right Lines A R, A S, the Point A will be ſollicited by three Powers, whereof two act according to the Directions A R, A S, and the other is the Force of the Gravity of the Weight B, acting according to the right Line A B perpendicular to the Earth; whence the Power R will be to the Force of Gravity as A C to A D, or as the Sine of the Angle D A E to the Sine of the Angle D E A or C A E; and the Power S will be to the Force of Gravity as E A to A D, or the Sine of the Angle C A D to the Sine of the Angle D E A or C A E; and the Power R will be to the Power S as the Sine of the Angle E A D to the Sine of the Angle C A D.

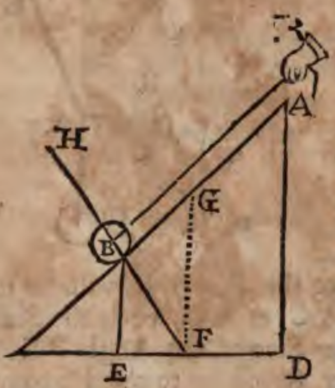


This Theorem, with its Corollaries, is the Foundation of the *New Mechanicks* publiſhed by M. *Varignion*; and from it likewiſe may immediately be deduced moſt of the mechanick Theorems contained in the famous Book of *Borelli*, *de Motu Animalium*, for by its help may the Force of the Muſcles be determined.

THEOR. XXXIV.

IF *an heavy Body* B *is incumbent on an inclined Plane, and is ſupported by a Power* R, *acting according to a Direction that is parallel to the Plane, ſo that it does not deſcend along that Plane; the Power* R *will be to the Weight of the Body* B *as the Sine of the Angle of Inclination to the Radius.*

THOUGH the Point where the heavy Body reſts againſt the Plane, draw A C perpendicular to the common Section of the Plane and the Horizon; from any Point of which A, let fall on the Plane of the Horizon the Perpendicular A D, and join C D; A C D will be [by Def. 6. El. 11.] the Angle of Inclination of the Plane and the Horizon, whoſe Sine is A D, C A being made the Radius. Now I ſay, A C is to A D as the Weight of the Body B to the Power R. For the Body B is ſollicited by three Powers acting according to different Directions, and placed *in æquilibrio*; whereof the firſt is the Force of Gravity acting according to the Direction B E perpendicular to C D; the ſecond is the Power R, drawing the Body according to the Direction B R parallel to A C; but the place of the third Power is ſupplied by the Reſiſtance of the Plane acting according to the Line B H perpendicular to it; for Re-action is always equal to Action, and is made towards the contrary Part: and ſince the Plane is preſſed perpendicularly by the moveable Body,



Body, according to the Direction B H, and that contrary Endeavour is equivalent to the Power ſollicting the Body according to B H; and ſince theſe three Powers are *in æquilibrio*, and the moveable Body is ſuſtained by them, if F G is drawn parallel to E B, meeting the right Line A G in G, the Power R will be to the Force of Gravity as B G to F G, [by the preceding Theor.] but by reaſon the Triangle C F G is rectangular, and F B a Perpendicular to its Baſe C G, it is [by 8 *El.* 6.] as B G to F G, ſo is F G to G C, and as F G to G C, ſo [by 4 *El.* 6.] will be A D to A C: wherefore the Power R is to the Force of Gravity as A D to A C, or as the Sine of the Inclination of the Plane to the Radius. Any Power therefore may be able to ſuſtain a heavy Body on an inclined Plane, provided the Power is to the Weight of the heavy Body as the Sine of the Inclination of the Plane to the Radius. *Q. E. D.*

Cor. 1. SINCE the Power R hinders the Deſcent of the heavy Body down the Plane A C, and is equivalent to its Momentum, whereby it endeavours to deſcend down the Plane; it follows that the Force of any heavy Body deſcending down an inclined Plane, is to the Force wherewith it endeavours to deſcend in the Perpendicular, as the Sine of Inclination of the Plane to the Radius.

Cor. 2. HENCE there may be aſſigned ſuch an Inclination of a Plane, that any, the ſmalleſt Power, may be able to ſuſtain or even raiſe upon that Plane any Weight, how great ſoever.

LECTURE XV.

Of the Deſcent of heavy Bodies on inclined Planes, and of the Motion of Pendulums.

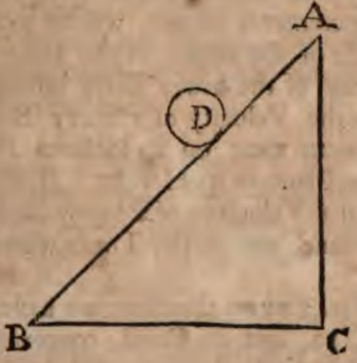
HAVING finiſhed what belongs to Motion in general, we ſhall now proceed to treat of thoſe Motions that ariſe from given Forces; in explaining of which, and reviewing the Phenomena thence ariſing, conſiſts the Buſineſs of the true Philoſophy. That therefore we may begin with what is moſt ſimple, we ſhall in the firſt place conſider that Force which is always directed toward the ſame Part uniformly, that is, every where with the ſame Tenour; and ſuch is commonly ſuppoſed the Force of Gravity. For though it is certain that the Force of Gravity is not every where the ſame, but at different Diſtances from the Center of the Earth, is reciprocally as the Squares of ſuch Diſtances; yet ſince the different Heights to which heavy Bodies can be thrown by us, are very ſmall, in compariſon of the vaſt Diſtance from the Center of the Earth, in ſo very little Difference of Heights, we may ſafely, and without any ſenſible Error, ſuppoſe the Force of Gravity to be every where the ſame.

WE ſhall therefore in this place treat of the Motion of heavy Bodies: but we ſuppoſe this Motion to be performed, either in Planes inclining to the Horizon, or in curve Superficies, ſuch as are ſpherical and cycloidal ones; or laſtly, in free and unreſiſting Spaces: concerning all which, we ſhall give the following Theorems.

THEOR. XXXV.

THE *Deſcent of a heavy Body upon any inclined Plane, is a Motion equably accelerated. And the Velocity that a heavy Body falling from Reſt along an inclined Plane, acquires in any given Time, is to the Velocity acquired in the ſame Time, by a heavy Body falling perpendicularly, as the Height of the Plane to its Length.*

LET A B be the inclined Plane on which the heavy Body D deſcends. [By Corol. 1. Theor. 34.] the Force wherewith a heavy Body endeavours to deſcend on an inclined Plane, is to the abſolute Force of Gravity, namely that whereby it would deſcend in the Perpendicular, in a conſtant Ratio, which is that of the Sine of the Inclination of the Plane to the Radius, or as the Height of the Plane to its Length; and therefore ſince the abſolute Force of Gravity of the Body D remains the ſame, the Force likewiſe wherewith it endeavours to deſcend on the Plane A B, will remain the ſame. That Force therefore will always act on the heavy Body D in the ſame Tenour; ſo that being applied in the ſame manner, it will, by the ſecond Law, always add equal Increments of Veloci-



ties, not otherwiſe than is done in heavy Bodies falling perpendicularly. The Deſcent therefore of heavy Bodies on an inclined Plane, is a Motion uniformly accelerated. *Q. E. D.*

MOREOVER, the Increments of the Velocities of heavy Bodies falling in the Perpendicular, and on an inclined Plane, which are produced in the ſame indefinitely ſmall Time, are to one another as the Forces wherewith they are produced: but the Forces are in a conſtant Ratio, namely, as the Length of the Plane A B to its Height A C; wherefore the Increments of the Velocities thence ariſing, will be in the ſame Ratio. And conſequentld, [by 12 *El.* 5.] the Sum of the Increments of one, will be to the Sum of the Increments of the other, in the ſame Ratio, that is, the Velocity of the heavy Body falling perpendicularly, is to the Velocity of a heavy Body deſcending in the mean time on an inclined Plane, as the Length of the Plane to its Height. *Q. E. D.*

Cor. 1. THE Velocities of a heavy Body falling on an inclined Plane, are as the Times wherein they are acquired.

Cor. 2. WHATEVER therefore we have demonſtrated in Theor. 12. and its Corol. concerning Motion uniformly accelerated, the ſame will hold true in the Deſcent of heavy Bodies on inclined Planes; namely, the Space paſſed over in a given Time by a heavy Body falling on an inclined Plane, computing from the beginning of Motion, will be the half of that, which in the ſame Time may be paſſed over by a Body moving uniformly, with the Velocity acquired at the laſt. Alſo the Spaces paſſed over, computing from the beginning of the Motion, are in a duplicate Ratio of the Times or Celerities; and the Celerities and Times are in a ſubduplicate Ratio of the Spaces paſſed over.

Cor. 3. HENCE alſo the Aſcent of a heavy Body along any inclined Plane, is a Motion uniformly retarded, as it is in the Aſcent of a Body in the Perpendicular, and it has juſt the ſame Properties.

SCHOLIUM.

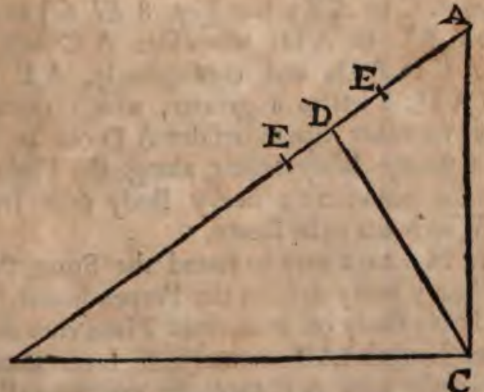
IF we have recourſe to Experiments, we ſhall find all things anſwer to our Reaſonings; and in Planes that are not too much inclined, it will be eaſy to make an Experiment, ſince Motions that are not very ſwift may be exactly meaſured: But it is the contrary in the perpendicular Deſcent, where the Swiftneſs of the Motion gives not time to make accurate Obſervations.

WE here ſuppoſe the Planes to be exactly ſmooth, and the Motion on them to be hindered by no Ruggedneſs or Unevenneſs.

PROBL. V.

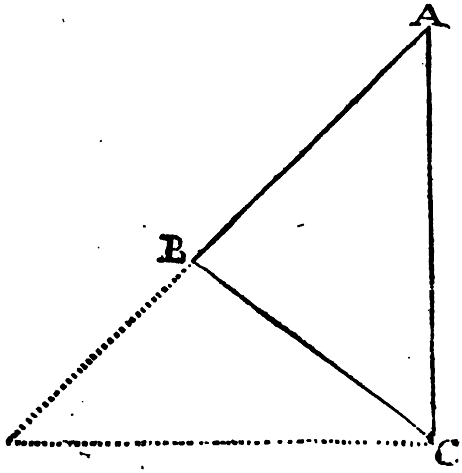
AN *inclined Plane being given, to aſſign the Part of it which a heavy Body will paſs over, in the Time whilſt another heavy Body ſhall have fallen through a given Space in the Perpendicular.*

LET AB be the inclined Plane on which the heavy Body deſcends from A; the Length is to be



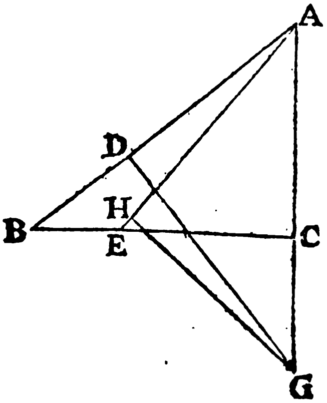
aſſigned in it, which will be paſſed over by the heavy Body falling along the inclined Plane: whilſt in the mean while another heavy Body falling perpendicularly, ſhall have run over the Space A C. From the Point C let fall the Perpendicular C D on A B meeting the Plane in D; then A D will be the Space paſſed over along the inclined Plane, in the Time wherein the heavy Body falls in the Perpendicular from A to C. For if A D is not, let A E be the Space deſcribed in the ſame Time that the heavy Body falls from A to C, which let be greater or leſs than A D. Draw the horizontal right Line C B. And becauſe by Theor. 12. in the Time that the heavy Body falls from A to C, or from A to E, it may run over twice the Length of A C, with an uniform Velocity, and equal to that which it acquires from falling to C, [as by the Cor. of the preceding Theor.] in the ſame Time a Length double of A E may be run over, with the Velocity that is acquired in E; the Velocity in C, by Theor. 6. will be to the Velocity acquired in E as twice A C to twice A E, or as A C to A E. But ſince A C, A E, are run over in the ſame Time, the Velocity in C [by the preceding Theor.] will be to the Velocity in E as A B to A C: wherefore A B will be to A C as A C to A E; but [by 8 *El.* 6.] as A B to A C, ſo is A C to A D: wherefore A C will be to A E as A C to A D; and conſequently, A E will be equal to A D, a leſs to a greater, which cannot be. Therefore no other Space beſides A D can be paſſed over by a heavy Body falling along the Plane A B, whilſt in the mean time a heavy Body falls from A to C. Which was to be ſhewn.

Cor. 1. HENCE may be found the Space through which a heavy Body falls in the Perpendicular, whilſt another heavy Body on an inclined Plane runs through any given Length A B: namely, if from the Point B to the right Line A B there be erected a Perpendicular



dicular BC, meeting the Perpendicular in C, then A C will be the Space ſought.

Cor. 2. If there are two or more inclined Planes AB, AE; and there is given the Space AD in the Plane AB, paſſed over by a heavy Body in any Time;



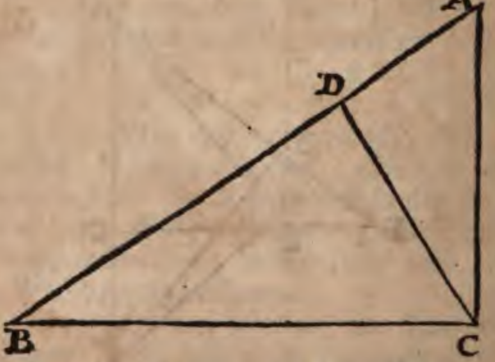
the Space that will be run through in the ſame Time by a heavy Body on the Plane AE, will be found by erecting from the Point D the Perpendicular D G, meeting with the Perpendicular in G: and by letting fall from G on A E the Perpendicular G H, meeting the Plane A E in H; then A H will be the Space ſought: for both the Spaces A D, A H, will be paſſed over in the Time that the heavy Body deſcends in the Perpendicular from A to G.

Cor. 3. FROM the Demonſtration of this Theorem it is manifeſt, that the Velocities acquired in the ſame Time by heavy Bodies falling in the Perpendicular and on inclined Planes, are as the Spaces paſſed over by thoſe Bodies.

THEOR. XXXVI.

THE *Time wherein the inclined Plane* A B *is paſſed over, is to the Time wherein the Perpendicular* A C *is run through, as* A B *the Length of the Plane, to* A C *the Length of the Perpendicular.*

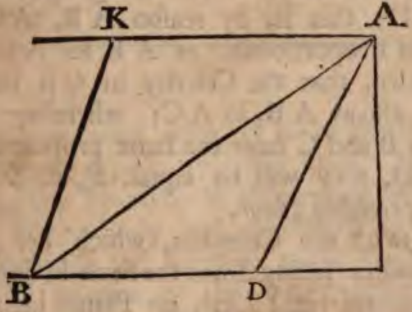
FROM C to A B let fall the Perpendicular C D; and the Time wherein A D is paſſed over, will be



equal to the Time wherein A C is paſſed over. But the Time wherein A B is paſſed over, is to the Time wherein A C is paſſed over, in a ſubduplicate

Ratio of A B to A D, [by Corol. 2. Theor. 35.] that is, by reaſon A B, A C, A D, are continual Proportionals, the Time wherein A B is paſſed over, is to the Time wherein A D or A C is paſſed over, as A B to A C. *Q. E. D.*

Cor. HENCE the Time wherein different Planes, A B, A D, B K, whoſe Height is the ſame, are paſſed over, are as the Lengths of the Planes: for the Time in A B is to the Time in A C, as A B to A C; and the Time in A C to the Time in A D, as A C to A D: wherefore by Equality the Time in A B will be to the Time in A D, as A B to A D.



THEOR. XXXVII.

THE *Celerities of heavy Bodies, on any inclined Plane, and in the Perpendicular, are equal, where the heavy Bodies ſhall deſcend from the ſame Height to the ſame Horizontal right Line.* See Fig. Theor. 36.

LET the inclined Plane be A B, and the Perpendicular A C. Draw the Horizontal right Line B C. I ſay, the Celerity acquired in the Point B, after the Deſcent through A B, will be equal to the Celerity acquired in the Point C, after the Fall through A C. From the Point C let fall the Perpendicular C D on

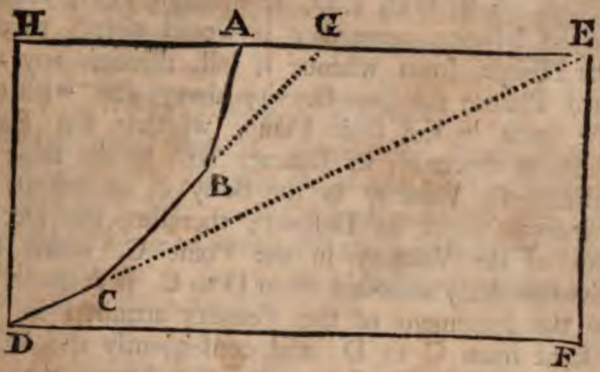
A B. Then A D will be the Space that is run over by the heavy Body falling in the Plane A B, in the Time that another heavy Body deſcends in the Perpendicular through A C: and [by Cor. 3. Probl. 5.] the Celerity in C is to the Celerity in D as A C to A D, or as A B to A C. But becauſe the Celerities acquired in falling along the ſame Plane are in a ſubduplicate Ratio of the Lengths run over by the heavy Body, the Celerity in B will be to the Celerity in D, in a ſubduplicate Ratio of the Length A B to the Length A D; that is, by reaſon A B, A C, A D, are continual Proportionals, as A B to A C: but it has been ſhewn, that the Celerity in C is to the Celerity in D, alſo as A B to A C; wherefore ſince the Celerities in B and C have the ſame proportion to the Celerity in D, they will be equal. *Q. E. D. See Fig. Cor. of the preceding Theor.*

Cor. HENCE the Celerities, which are acquired by heavy Bodies falling from the ſame Height to the ſame Horizontal right Line, on Planes inclined after any manner, are equal amongſt themſelves; for both the Celerities, namely, that which is acquired in the Point B, after the Deſcent through A B or B K; and that which is acquired in the Point D, after the Deſcent through A D; is equal to the Celerity acquired in the Deſcent of the heavy Body from A to C.

THEOR. XXXVIII.

IF *from the ſame Height a moveable Body deſcends with an uninterrupted Motion through any Number of contiguous Planes* A B, B C, C D, *it will always at the concluſion acquire the ſame Velocity; namely, that which is equal to what is acquired by falling perpendicularly from the like Height.*

THROUGH A and D draw the Horizontal right Lines H E, D F, and produce the Planes B C, C D,

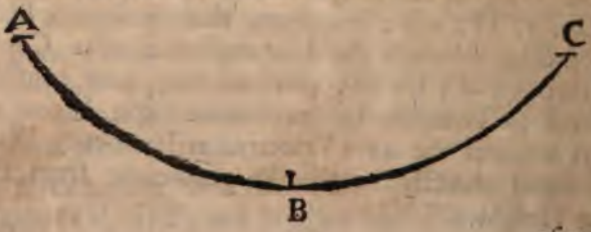


that they may meet with HE in the Points G and E. [By Corol. Theor. 37.] the ſame Celerity is acquired in the Point B, by deſcending through A B, as if the heavy Body had deſcended through B G: but we ſuppoſe the Flexure at the Point B, not to hinder the Motion of the heavy Body, but only to change its Direction; ſo that in the Point C the ſame Velocity will be acquired by deſcending through A B, B C, as if it had deſcended through G C. But by deſcending through G C, the ſame Celerity is acquired, as the heavy Body would have gotten by falling through E C; ſo that ſince the Flexure at C is not ſuppoſed to diminiſh the Velocity of the heavy Body, it will have the ſame Velocity in D, as if it had deſcended through the Plane E D, or the Perpendicular E F. *Q. E. D.*

Cor. 1. HENCE it appears, that a moveable Body deſcending through the Circumference of a Circle, or through any Curves, (for we may here conſider Curves as compounded of infinite right Lines) always acquires the ſame Velocity, as if a heavy Body deſcended directly in the Perpendicular from the ſame Height.

Cor. 2. But if a heavy Body, after its Deſcent through A B, B C, C D, or through H D, ſhould turn its Motion upwards; it would aſcend to the ſame Height from whence it fell, through any inclined Planes: for ſince Gravity always acts with the ſame force in the ſame Plane, whether the Body aſcends or deſcends, its Efficacy will be the ſame to diminiſh the Velocity in the Body in its Aſcent, as to increaſe it in its Deſcent; therefore the Decrement of the Velocity in the Point C, whilſt the moveable Body aſcended from D to C, is as great, as was the Increment of the Velocity acquired in the Deſcent from C to D: and conſequently the Velocity in C will be the ſame after the Aſcent through C D, as it was before in the ſame Point, after the Deſcent through AB, BC. In like manner, the Velocity in B after the Aſcent through C B is the ſame with the Velocity acquired in the Deſcent through A B or B G. So alſo Gravity will take as much from the Velocity of the Body in aſcending through BA, as was acquired in the Deſcent through AB; and in the Points that are of an equal Height, the Velocity of the Body will be always the ſame, but the Velocity in the beginning of the Deſcent, namely, in the Point A, was nothing: ſo that by aſcending, at that Point A all the Velocity is taken away; which Point therefore will be the Boundary to which the moveable Body by aſcending will arrive.

Cor. 3. If a moveable Body deſcends through any Superficies AB to its loweſt Point B, and afterwards

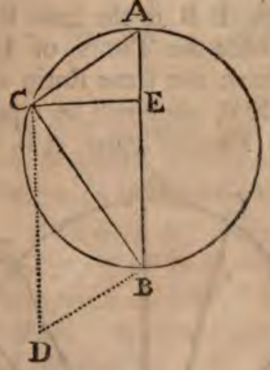


aſcends by the Velocity acquired in the Fall, through the ſimilar and equal Superficies BC; it will aſcend and deſcend in equal Times through equal Spaces.

THEOR. XXXIX.

If *from the higheſt Point* A, *or loweſt* B, *of a Circle perpendicular to the Horizon, be drawn any two inclined Planes* AC, BC, *to the Circumference; the Times of the Deſcents through thoſe Planes, will be equal to the Time wherein a heavy Body falls perpendicularly through the Diameter.*

Let a heavy Body fall from A to C, on the Plane AC; I ſay, the Time of Deſcent through AC is equal to the Time of Deſcent through the Diameter AB. For the Angle ACB in the Semicircle is a right one, [by 31 *El.* 3.] whence ſince from the

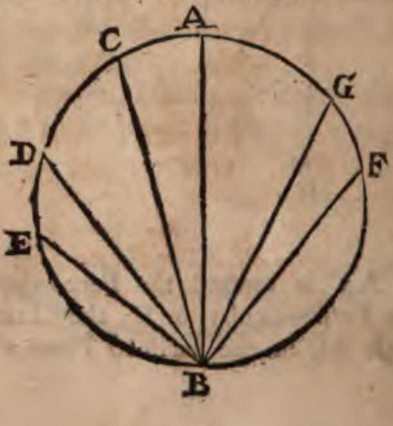


Point C to AC is erected the Perpendicular CB, meeting the Perpendicular in B; the Time of Deſcent through AC in the inclined Plane will be [by

Corol. 1. Probl. 5.] equal to the Time of the Fall through AB in the Perpendicular. I ſay alſo, the Time of Deſcent through CB will be equal to the ſame Time through AB. Draw CD parallel to AB, and DB to AC, and [by 34 *El.* 1.] CD will be equal to AB: and by reaſon of the Angle ACB being right in the Semicircle, the Angle CBD will be a right one. Wherefore ſince from the Point B, upon CB, is erected at right Angles BD meeting with the Perpendicular in D; the Time of Deſcent through CB will be [by Corol. Probl. 5.] equal to the Time of Deſcent through CD: but CD is equal to AB, whence the Time of Deſcent through CB will be equal to the Time of Deſcent through AB.

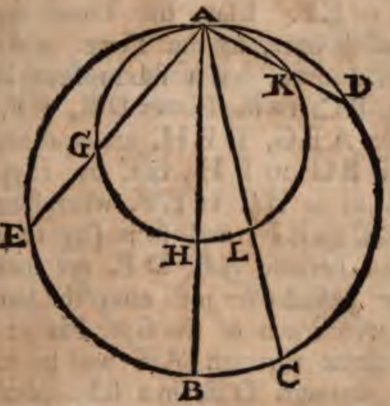
THE ſame thing may be otherwiſe ſhewn thus. The Time of Deſcent through AB is to the Time through EB, in a ſubduplicate Ratio of AB to EB, that is, (by reaſon AB, BC, EB, are continual Proportionals) as AB to BC, or BC to BE; but [by Theor. 36.] the Time of Deſcent through BC is to the Time through EB in the ſame Ratio of BC to EB: wherefore ſince the Times of Deſcent through AB and BC have the ſame Ratio to the Time of Deſcent through EB, they will be equal. *Q. E. D.*

Cor. 1. IF the Perpendicular AB is drawn, and



upon the Diameter A B is deſcribed a Circle, all the Planes drawn from the Point B, or from the Point A to the Circumference of the Circle, will be run over in the ſame Time: namely, the Planes A B, C B, D B, E B, F B, G B, will be paſſed over in the ſame Time.

Cor. 2. If in the higheſt Point A, any number of Circles, A B D, A G K, touch each other; and any number of Planes proceed from A, as A B, A C, A D,

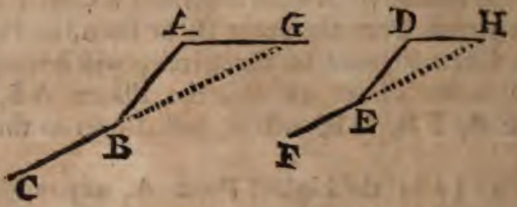


A E, interſecting the Circles: the Parts G E, H B, L C, K D, will be paſſed over in an equal Time, if the Motion is begun at the higheſt Point A.

THEOR. XL.

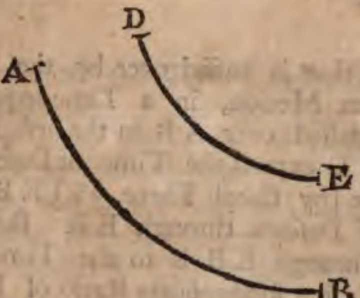
If *two heavy Bodies deſcend in two or more Planes, inclined alike, and proportional; the Times ſpent in running through them, will be in a ſubduplicate Ratio of the Lengths of the Planes.*

Let any heavy Body run through the Planes A B, B C, but another heavy Body run through the Planes D E, E F, inclined alike to the Horizon, and proportional; that is, that the Angles B A G, E D H, are



are equal, as are alſo B G A, E H D; and A B is to B C as D E to E F. I ſay, the Time wherein A B, B C, are paſſed over, to the Time wherein D E, E F are paſſed over, has a ſubduplicate Ratio of the Planes A B, B C, to the Planes D E, E F. By reaſon the Triangles A B G, D E H, are equi-angular, A B is to D E as B G to E H; but, by Hypotheſis, as A B to D E ſo is B C to E F, wherefore as B G to E H ſo is B C to E F; and ſo is [by 12 *El.* 5.] G C to H F. But becauſe A B, D E, are inclined alike, they will be paſſed over juſt after the ſame manner, as if they were Parts of the ſame Plane: ſo that the Time of Deſcent through A B will be to the Time of Deſcent through D E, in a ſubduplicate Ratio of A B to D E; and the Time of Deſcent through G C is to the Time of Deſcent through H F, in a ſubduplicate Ratio of G C to H F, or in a ſubduplicate Ratio of A B to D E. But the Time of Deſcent through G B is to the Time of Deſcent through H E, in a ſubduplicate Ratio of G B to H E, or A B to D E; ſo that [by 19 *El.* 5.] the Time of Deſcent through B C, after the Deſcent from G or A, is to the Time of Deſcent through E F after the Deſcent from H or D, in a ſubduplicate Ratio of A B to D E; that is, as the Time of Deſcent through A B to the Time of Deſcent through D E. So that [by 12 *El.* 5.] the Time of Deſcent through A B, B C, will be to the Time of Deſcent through D E, E F, as the Time of Deſcent through AB, to the Time of Deſcent through DE, or in a ſubduplicate Ratio of AB to DE: but by reaſon AB is to DE as BC to EF, AB will be to DE as AB, BC to DE, EF. So that the Time of Deſcent through AB, BC, will be to the Time of Deſcent through DE, EF, in a ſubduplicate Ratio of AB, BC, to DE, EF. *Q. E. D.* The ſame thing, in like manner, might be ſhewn, if there were more Planes on both Parts inclined and proportional; whence the Propoſition is manifeſt.

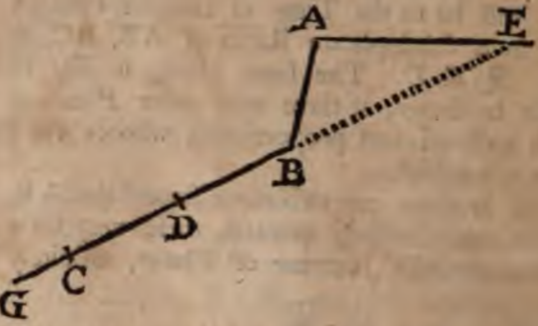
Cor. If there are two curve Superficies AB, DE, ſimilar and ſimilarly poſited, theſe very little differ from an infinite Number of Planes, infinitely ſmall, and proportional, and inclined alike to one another; ſo that the Time of Deſcent through the Superficies AB, will be to the Time of Deſcent through the Superficies DE in a ſubduplicate Ratio of AB to DE.



PROBL. VI.

The *Space* AB *being given in any inclined Plane, and which is paſſed over in a given Time by a heavy Body falling from reſt; to find a Space in another contiguous Plane* BG, *that will be paſſed over in an equal Time, by the heavy Body continuing its Motion along this ſecond Plane.*

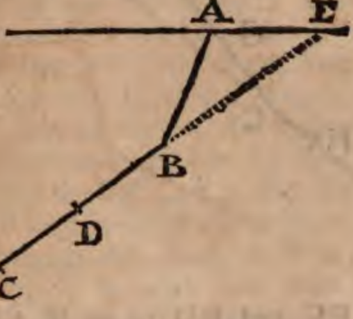
THROUGH A draw the Horizontal right Line A E, and produce B G to E; make B D equal to A B, and take E C a third proportional to the right Lines E B, E D. Then B C will be the Space in the ſecond Plane that is paſſed over by the heavy Body continuing its Motion, in a Time equal to that, wherein it paſſed over A B in the firſt Plane. For let A B or B D expreſs the Time of Deſcent through A B, whence [by Corol. Theor. 36.] E B will expreſs the Time of Deſcent through E B. But the Time of Deſcent through E B is to the Time of Deſcent through E C, in a ſubduplicate Ratio of E B to E C, that is, as E B to E D; but E B is the Space that is paſſed over in the Time as E B; ſo that E C will be the Space that is paſſed over in the Time as E D, and conſequently B C is the Space that is paſſed over in the Time as D B or A B, after the Fall from E or A. Which was to be found out.



PROBL. VII.

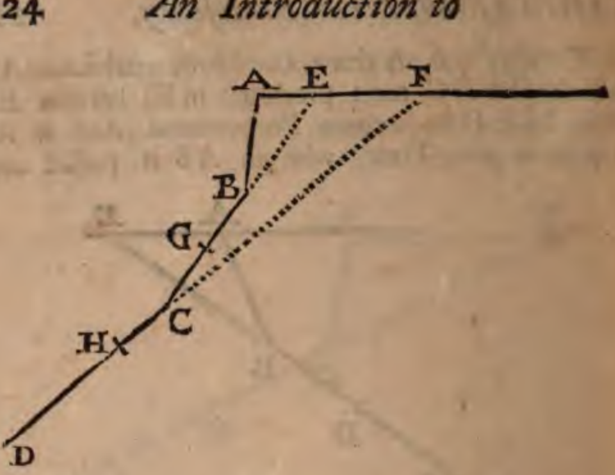
THE *Space* A B *being given in an inclined Plane, which is paſſed over in a given Time by a heavy Body falling from reſt; as alſo the Space* B C *in another contiguous Plane, wherein the heavy Body continues its Motion: to find the Time wherein that given Space* B C *will be paſſed over.*

THROUGH A draw the Horizontal Line AE, which let BC meet produced in E: betwixt EB, EC, let ED be a mean Proportional. And if AB repreſents the Time wherein AB is paſſed over, BD will repreſent the Time ſought, wherein BC will be paſſed over. For the Time of Deſcent through AB is to the Time of Deſcent through EB, as AB to EB; ſo that EB will expreſs the Time wherein the heavy Body will fall through EB: but the Time of Deſcent through EB is to the Time of Deſcent through EC, in a ſubduplicate Ratio of EB to EC; or, by reaſon EB, ED, EC, are continual Proportionals, as EB to ED: but EB is as the Time of Deſcent through EB, whence DB will be as the Time of Deſcent through BC. And conſequently the Time of Deſcent through AB, will be to the Time of Deſcent through BC, as AB to BD. *Q. E. I.*



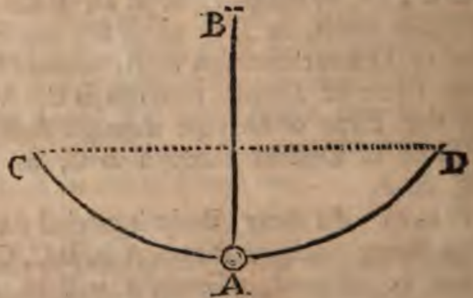
Cor. HENCE if a heavy Body is carried ſucceſſively through many inclined Planes AB, BC, CD, the Time may be aſſigned, wherein it will be moving through each of them: for produce BC, CD, that they may meet with the Horizontal right Line drawn through A, in the Points E and F, and let EG be a mean Proportional betwixt EB and EC; as FH

betwixt



betwixt F C and F D: And if A B expreſſes the Time of Deſcent through A B, B G will expreſs the Time of Deſcent through B C, and C H will expreſs the Time of Deſcent through C D.

Def. If any heavy Body A, is hung by a fine String moveable about a Center B; the heavy Body, together with the String, is called a Pendulum. Now if the Pendulum is ſwung about B,



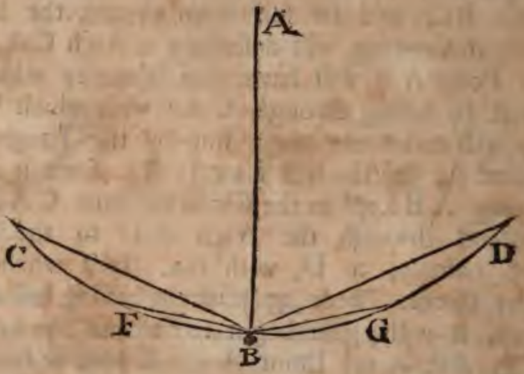
ſo that the heavy Body may deſcribe the Arch C A D, the ſame Motion will happen to this heavy Body, as would to any Body deſcending by its Gravity along the ſpherical Superficies C A D, if that Superficies was perfectly hard and ſmooth. For we ſuppoſe the Motion about the Point B to be intirely free, and we abſtract from the Conſideration of the Air's Reſiſtance, which in heavy Pendulums is very inconſiderable: if the Pendulum is carried to the Situation B C, and let fall from thence, the heavy Body in deſcending will deſcribe the Arch C A, and in the Point A it will have that Velocity which is acquired by falling through C A: with which Velocity it will endeavour to go out by the Tangent in the Point A, [by the firſt Law.] But ſince it is by the String A B kept in the Circumference C A D, it will aſcend through the Arch A D to the ſame Height, namely, to D, with that from whence it fell, [by Corol. Theor. 37.] where having loſt all its Velocity, it will again begin to deſcend by its own Gravity; and in the Point A it will acquire the former Velocity, wherewith it will aſcend to C: and ſo by aſcending and deſcending, it will perform continual Vibrations in the Circumference C A D. And if the Motions of Pendulums met with no Reſiſtance from the Air, and if there was no Friction at the Center of Rotation B, the Vibrations of Pendulums would never ceaſe; but from theſe Cauſes, the Velocity of the Pendulum in the Point A is a little, tho' very inſenſibly diminiſhed in every Vibration; whence it falls out, that the Ball of the Pendulum does not return preciſely to the ſame Point, but the Arches which it deſcribes conſtantly become ſhorter, till at length they grow inſenſible.

THEOR. XLI.

THE *ſmall Vibrations of the ſame Pendulum, tho' they ſhould be unequal, are almoſt and to Senſe performed in the ſame Time.*

LET A B be a Pendulum, which by Vibrating deſcribes the unequal Arches C B D, F B G; I ſay, the

Times taken up in deſcribing them are nearly equal: or the Vibration in the Arch CBD will be nearly performed in a Time equal to that, wherein the Vibration in the Arch FBG will be performed, if the Arches CB, FB, are not too large. Draw the Subtenſes CB, FB, DB, GB, and becauſe the Arches are ſuppoſed very ſmall, they differ not much either in Length or Declivity from their Subtenſes or Chords: and conſequently the heavy Body takes up equal Times, whether it is carried along the Arches CB, FB, or the Subtenſes of thoſe Arches; but the Times of Deſcent through the Subtenſes of Arches are equal, [by Theor. 37.] wherefore the Times of Deſcent through the Arches CB, FB, will be nearly equal; and therefore the Doubles of theſe Times, namely, wherein the unequal Arches CBD, FBG, are deſcribed by vibrating, will be alſo equal. Wherefore the Vibrations of the ſame Pendulum, though running out into unequal Arches, are to Senſe at leaſt of equal Duration. *Q. E. D.*

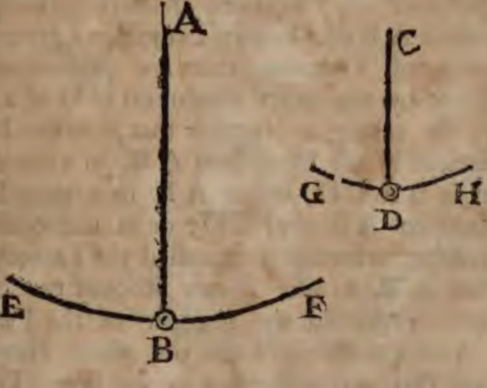


EXPERIENCE agrees with this Theorem; for two Pendulums of equal Lengths being put in motion, and whereof one deſcribes much greater Arches than the other, they will nearly obſerve equal Times in their Vibrations; ſo that in a hundred Vibrations there will ſcarce be the Difference in Time of one Vibration.

THEOR. XLII.

THE *Durations of the Vibrations of two Pendulums, deſcribing ſimilar Arches, are in a ſubduplicate Ratio of the Lengths of the Pendulums.*

LET there be two Pendulums AB, CD, vibrating in ſimilar Arches EBF, GDH; the Time of a Vibration of the Pendulum AB will be to the



Time of a Vibration of the Pendulum CD, in a ſubduplicate Ratio of the Length AB to the Length CD. For becauſe the Arches EB, GD, are ſimilar, and poſited alike, the Time of Deſcent through EB will be [by Theor. 39.] to the Time of Deſcent through GD, in a ſubduplicate Ratio of EB to GD; but the Time of Deſcent through EB is half a whole Vibration in the Arch EBF, as the Time of Deſcent through GD is half a Vibration in the Arch GDH; ſo that the Time of Vibration of the

Pendulum in the Arch E B F will be to the Time of Vibration of the Pendulum in the Arch G D H, in a ſubduplicate Ratio of E B to G D; that is, by reaſon the Arches E B, G D, are ſimilar, in a ſubduplicate Ratio of the Semidiameter A B to the Semidiameter C D, or in a ſubduplicate Ratio of the Length of the Pendulum A B to the Length of the Pendulum C D. *Q. E. D.*

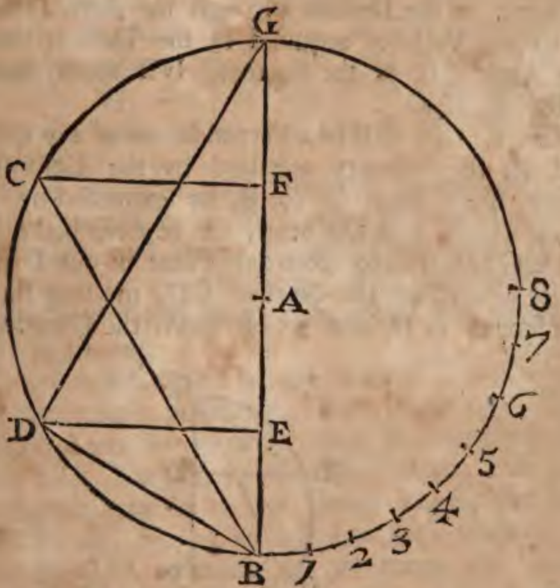
Cor. THE Lengths of Pendulums are in a duplictae Ratio of the Times wherein the Vibrations are performed.

SINCE the Durations of the Vibrations are reciprocally as the Number of Vibrations performed in the ſame Time, there may be eaſily found from a given Number of Vibrations that are performed by a Pendulum A B of a known Length in a given Time, the Number of Vibrations that are performed in the ſame Time by any other Pendulum C D of a known Length; by taking a Number that is to the Number of Vibrations of the Pendulum A B, in a ſubduplicate Ratio of A B to C D, or as A B to a mean Proportional betwixt A B and C D; or as the ſquare Root of a Number whereby is expreſſed the Length of the Pendulum A B, to the ſquare Root of the Number whereby is expreſſed the Length of the Pendulum C D. And alternately from the given Number of Vibrations that are performed in the ſame Time by two Pendulums A B, C D, and the Length of one, namely, A B being given; there will be given the Length of the other C D, *viz.* by making the Square of the Number of Vibrations of the Pendulum C D to the Square of the Number of Vibrations of the Pendulum A B, ſo the Length of A B to C D the Length ſought.

THEOR. XLIII.

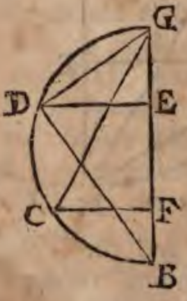
THE *Velocity of a Pendulum, in its loweſt Point, is as the Subtenſe of the Arch which it has deſcribed in its Deſcent.*

LET AB be a Pendulum, which by its Motion deſcribes the Circle BDCG; I ſay, the Velocity acquired by falling from D to B, is to the Velocity acquired in B by falling from C to B, as the Chord of the Arch BD, to the Chord of the Arch BC. Through the Points D, C, draw the Horizontal right Lines DE, CF; and the Velocity the heavy Body acquires by deſcending through EB will be to the Velocity it will acquire in its Deſcent through GB, in a ſubduplicate Ratio of EB to GB, that is, by reaſon EB, DB, GB, are continual Proportionals, as DB to GB. By the ſame reaſon, the Velocity acquired by the moveable Body, in falling through GB, is to the Velocity acquired in the Fall through FB, as GB to CB. Wherefore by Equality, the Velocity acquired, in the Deſcent of the heavy Body through EB, will be to the Velocity acquired



in the Deſcent through F B, as D B to C B; but the Velocity acquired in the Deſcent through the Arch D B, is the ſame with the Velocity acquired in the Perpendicular through E B; and the Velocity acquired in the Deſcent through the Arch C B, is the ſame with the Velocity acquired by the Deſcent in the Perpendicular through F B. Wherefore the Velocity acquired in the Deſcent through the Arch D B, will be to the Velocity acquired in the Deſcent through the Arch C B, as the Subtenſe D B to the Subtenſe C B. *Q. E. D.*

Cor. 1. LET G B be a Perpendicular of any Length, and let the Velocity acquired by the Deſcent of a heavy Body from G to B, be expreſſed by G B; upon which, as a Diameter, let be deſcribed a Semicircle G D C B, and from any Point of the Diameter E let be erected the Normal E D, meeting the Circumference in D, and let be drawn the Chord G D;

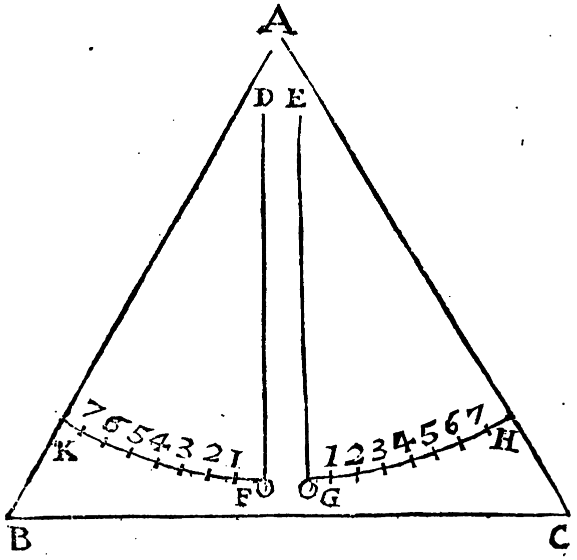


this will be as the Velocity acquired by the heavy Body in falling from the Height G E: for by reaſon B G, G D, G E, are continual Proportionals, the Ratio of B G to G D, will be ſubduplicate of the Ratio of B G to G E, ſo that B G will be to G D, as the Velocity acquired by falling from the Height G B, to the Velocity acquired by falling through G E. In like manner, the Velocity acquired by falling through G B, is to the Velocity acquired by the Fall through G F, as G B to G C: ſo that the Velocities acquired by heavy Bodies, in falling through the Heights G E, G F, are as the Chords G D, G C.

Cor. 2. If the Arches B 1, B 2, B 3, *&c.* are taken in ſuch a manner, that their Subtenſes are as 1, 2, 3, *&c.* reſpectively; and a certain Force acting on it, ſhould compel the Pendulum to move upwards through the Arch B 1, but another Force make it move through the Arch B 2, and another through the Arch B 3, the Velocities of the Pendulum put in motion by theſe Forces, will be in the point B, reſpectively as 1, 2, 3.

By the help of this Theorem, various Velocities in any given Ratio may be communicated to a moveable Body; and other Velocities acquired by the Percuſſion of another Body, may be compared amongſt themſelves, and with others that were given at the beginning.

Let A B C be a Triangle made of Wood, wherein near the Angle A let be taken two Points D, E, whoſe Diſtance let be ſuch, that the two Pendulums D F, E G, hanging freely from thence, may touch each other; and with the Centers D, E, and Interval D F, or E G, let be deſcribed the Arches of Circles F K, G H, in which let be taken the Portions F 1, G 1, F 2, G 2, F 3, G 3, F 4, G 4, *&c.* in ſuch a manner, that their Subtenſes may be as 1, 2, 3, 4, *&c.* reſpectively: and if the heavy Body F be raiſed up to the Point 5 in the Arch K F, but G to the Point 3 in the Arch G H, and are both let down at once, [by Theor. 41.] they will arrive in the ſame Time at the loweſt Points, and the Velocities wherewith they will ſtrike one another, will be as 5 and 3; but if after the Stroke the moveable Body G aſcends to 5 in the Arch G H, and the moveable Body F aſcend to 3 in the Arch F K, the Velocities of the move-



able Bodies F and G will be reſpectively as 3 and 5, and towards the contrary Parts. After this manner it will be eaſy to try by Experiments the Rules of Motion as well in hard as elaſtick Bodies, which we have demonſtrated in the 13th and 14th Lectures.

SINCE the very ſmall Vibrations of theſe Pendulums are nearly of equal Duration, altho' the Arches they deſcribe are unequal; hence the great Monſ. *Huygens* has ſhewn the vaſt Uſe of Pendulums to regulate the Motions of Clocks. For although *Gallileo*, the Author of this Science, firſt made uſe of Pendulums in Aſtronomical and Philoſophical Obſervations, which require an accurate Meaſure of Time; yet *Huygens* firſt adapted Pendulums to Clocks, and proved by Experience that ſuch Clocks were far more correct than the former, which were regulated by Horizontal Ballances. From that time theſe Pendulum

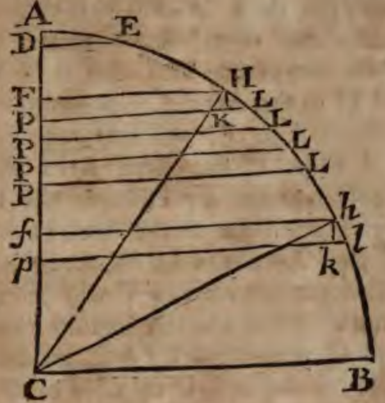
Clocks have been in common uſe, whereof ſome have been ſo elaborately made, that they give the Meaſure of Time much more correct than the Sun, which ſhews only the apparent or relative Time, but not the true and abſolute: whence it happens, that Pendulum Clocks ſhew at Times the Hour different from the apparent one, and ſometimes exceeding the Time ſhewn by a Sun-Dial by fifteen or ſixteen Minutes, and ſometimes falling as much ſhort of it in Time: nor does the Sun and Pendulum Clock ſhew the ſame Point of Time, except four times a year.

ALTHOUGH the Vibrations of the ſame Pendulum (notwithſtanding it deſcribes unequal Arches) are nearly and to Senſe of equal Duration; yet ſince they are not altogether and geometrically ſuch, but the greater are a ſomewhat longer time in deſcribing than the leſs, and the Vibrations differ from each other a ſmall Quantity in Time; from many of theſe very ſmall Differences at length there do ariſe a ſufficient large Difference, and this is found to be ſo by Experience: For if, as it ſometimes happens in cold Weather, the Wheels are clogged, ſo that they communicate a leſs Force to the Pendulum, the Vibrations become leſs, and the Time ſhewn by the Clock is too faſt; if the Wheels are too glibe, and force the Pendulum to deſcribe larger Arches, then the Times are pointed out too ſlowly. And from ſome late Experiments mentioned in the *Philoſophical Tranſactions*, it appears that the Pendulum of a Clock placed *in vacuo*, the Reſiſtance of the Air being thus removed, performed its Vibrations in larger Arches, and took up more Time in each Vibration. Wherefore that the Vibrations of a Pendulum might be reduced to an exact Equality, and that its Reciprocations, whether performed in greater or leſs Arches, might be made in equal Times; Monſ. *Huygens* found out a Method, whereby the Ball of a Pendulum may be always carried along the Arch of a Cycloid. We ſhall now demonſtrate in the following Thorems, that the Times of Deſcent through any Arches of the ſame Cycloid to its loweſt Point, which is ſuppoſed to be its Vertex, are equal to one another; ſo that if the Ball of a Pendulum is always moved in the Arch of a Cycloid, the Times of its Vibrations will be exactly equal amongſt themſelves, whether the Pendulum deſcribes greater or leſs Arches.

THEOR. XLIV.

If with the Center C, *and any Interval* C A, *there is deſcribed the Quadrant of a Circle* AHB, *and in the right Line* AC *a moveable Body deſcends with this Law, that its Velocity in any Point* P *ſhall be always as* P L, *the Sine of the Arch* A L; *the Time wherein the Body deſcends from* A *to* C, *will be equal to the Time wherein it might run through the Circumference* AHB, *with a uniform Velocity as* C B, *which is acquired at the laſt by the falling Body. Beſides, the Time of the Fall through any Space* A F, *will be to the Time of the Fall through the Space* A *p, as the Arch* A H *to the Arch* A *l*; *and the Force wherewith the Body is accelerated in any Place* F, *will be as* F C, *the Diſtance of that Place from the Center.*

Let the Circumference A B be diſtinguiſhed into innumerable infinite ſmall Parts L, L, L, L, and let be drawn FH, PL, P*l*, perpendicular to A C; join HC, and let H K be perpendicular to P L. Becauſe the Triangles F H C, K H L, are equi-angular, (for beſides the Angles at F and K being right ones, the Angle F H C is equal to the Angle K H L, the Angle K H C being the Complement of both to a right Angle) F H will be to H C as K H or F P to H L: but [by Hypotheſis] F H is as the Velocity of the moveable Body in the Point F, namely, wherewith the infinitely ſmall Line FP is run over, and CH or CB is as the Velocity which is at laſt acquired in falling, when the Body ſhall have come to C, ſo that it will be as the Velocity wherewith the



Arch HL will be deſcribed. The Velocity therefore of the Body deſcending through the infinitely ſmall Line FP, will be to the Velocity of the Body that is moved through the Arch HL, as FP to the Arch HL. Wherefore ſince the Velocities are proportionable to the Spaces paſſed over, the Times wherein thoſe Spaces are paſſed over will be equal. After the ſame manner it may be demonſtrated, that any other Particle LL of the Circumference may be deſcribed with the Velocity CB, in the ſame Time wherein the correſponding infinitely ſmall Line PP in the Perpendicular will be deſcribed with the correſponding Velocity PL; and conſequently by Compoſition, the moveable Body will deſcend through all the infinitely ſmall Lines PP, that is, through the whole AC, in the ſame Time wherein all the Arches LL, or the whole Circumference AHB is paſſed over, with a uniform Velocity as CB. *Q. E. D.*

BESIDES the Time wherein the moveable Body deſcends from A to F, is equal to the Time wherein the Arch AH is paſſed over; and the Time wherein it deſcends from A to p, is equal to the Time wherein the Arch A l is deſcribed; but the Time wherein the Arch A H is paſſed over, is to the Time wherein the Arch A l is paſſed over, (ſince they are both deſcribed with the ſame Velocity) as the Arch A H to the Arch A l; wherefore the Time of Deſcent from A to F, will be to the Time of Deſcent from A to p, as the Arch A H to the Arch A l; and conſequently by Diviſion the Time of Deſcent through F p will be as the Arch H h. *Q. E. D.*

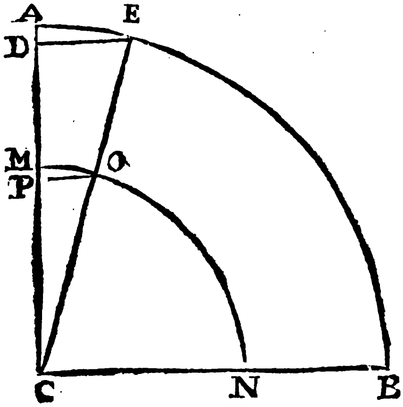
MAKE the Arches H L, $h\,l$, equal; whence the Time of Deſcent through F P will be equal to the Time through $f\,p$: and by reaſon the Triangles K H L, F H C, as alſo $k\,h\,l$, $f\,h$ C, are ſimilar; K L will be to H L or $h\,l$, as F C to C H or C h: Alſo $h\,l$ is to $k\,l$ as C h to C f; and conſequently, by Equality, K L will be to $k\,l$ as C F to C f: but K L is as the Increment of the Velocity acquired whilſt the Body paſſed over F P, and $k\,l$ is as the Increment of the Body's Velocity, whilſt it paſſed over in an equal Time the infinite ſmall Line $f\,p$; but the Forces wherewith the Body is accelerated in the Places F and f, are as the Increments of the Velocities generated in equal Times; the accelerating Forces therefore of the moving Body in the Places F and f will be as the right Lines K L, $k\,l$; that is, the force wherewith a Body is urged in F, is to the force wherewith a Body is urged in f, as the Diſtance G F to the Diſtance C P. The accelerating Forces therefore are in any Places as their Diſtances from the Center. *Q. E. D.*

Cor. Hence converſly if a moveable Body in deſcending from A to C is urged by a Force which is as its Diſtance from the Center; and that Force at the beginning of Motion is expreſſed by the right Line DE, the Arch A E being taking infinitely ſmall, the Velocities of the ſame Body in any Places F, *f*, will be expreſſed by the Sines F H, *f h*, and the Times by the Arches A H, A *h*, and the Increments of the Velocities; or if the Arches increaſe equally, the accelerating Forces will be expreſſed by the Increments of the Sines.

THEOR. XLV.

I F *a moveable Body be ſollicited in the right Line* A C *towards the Point* C, *with Forces that are proportionable to the Diſtances from the Point* C, *from whatever Height it falls, it will arrive at the Point* C *always in the ſame Time; and that Time is to the Time wherein a moveable Body may paſs over the ſame way, with a Velocity that is uniform and equal to that which is acquired at laſt by falling, as half the Circumference of a Circle to its Diameter.*

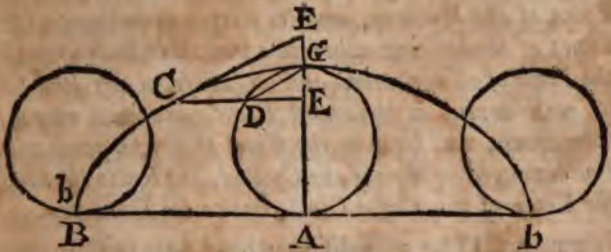
L E T two moveable Bodies be let fall at once from the Points A and M, and let both be ſollicited by Forces that are proportionable to their Diſtances from the Point C; I ſay, both the Bodies will arrive



at the point C in the ſame Time. With the Center C, and Intervals C A, C M, let be deſcribed two Quadrants of Circles A B, M N, and let the Force whereby the Body A is urged, or, which is the ſame thing, its Velocity at the beginning of Motion be repreſented by D E, the Sine of the infinitely ſmall Arch A E; it is manifeſt from the Corol. of the preceding Theor. that its Velocity, after the Fall to C, will be repreſented by the right Line C B. But, by Hypotheſis, the Force wherewith the moveable Body is accelerated in A, is to the Force wherewith the moveable Body is accelerated in M, as C A to C M, or as D E to P O, by reaſon the Arches A E, M O, are ſimilar: wherefore if D E expreſſes the Velocity of the moveable Body at the beginning of the Fall from A, P O will expreſs the Velocity of the moveable Body at the beginning of the Fall from M; and conſequently [by the ſame Corol.] C N will expreſs the Velocity of the Body in C after its Fall through M C. Beſides, the Time of the Fall from A to C, is equal to the Time wherein may be deſcribed the Circumference A B, with a uniform Velocity as C B; and the Time of the Fall from M to C, is equal to the Time wherein is deſcribed the Circumference M N with a Velocity as C N. But the Time wherein is deſcribed the Circumference A B with the Velocity C B, is equal to the Time wherein is deſcribed the Circumference M N with the Velocity C N, (by reaſon it is A B : M N : : C B : C N, namely, the Spaces paſſed over are proportionable to the Times) wherefore the Time of the Fall from A to C, will be equal to the Time wherein the Body deſcends from M to C. *Q. E. D.*

THE Time wherein the moveable Body paſſes over the right Line A C, with the Velocity C B, is to the Time wherein it runs through the Arch A B with the ſame Velocity, as the right Line A C to the Arch A B, or as its Double to the Double of this; that is, as the Diameter of the Circle to half its Circumference: but the Time of Deſcent through the Arch A B is equal to the Time of Deſcent to C; whence the Time wherein a Body is carried through the right Line A C with a Velocity as C B, will be to the Time of the Fall to C, as the Diameter of the Circle to half the Circumference. *Q. E. D.*

Defin. If a Circle inſiſting on the right Line B *b*, and touching it in its Point *b*, is ſuppoſed to revolve upon that right Line, and by its Circumference being continually applied to it, to meaſure a right Line B A *b* equal to that Circumference, till the Point *b* being carried aloft, and ſo deſcribing in its Motion the Curve B G *b*, and having finiſhed its Circuit, it again touches the right Line B A *b* in the Point *b*; the Curve deſcribed by the Motion of the Point is called a Cycloid. And the Figure B G D A B is called the Figure of the Cycloid; the right Line G A biſecting the Baſe perpendicularly, the Axis of the Cycloid; and the Point G the Vertex of the Cycloid. The Circle is named the generating Circle, and its Point *b* the deſcribing Point.



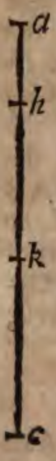
LEMMA.

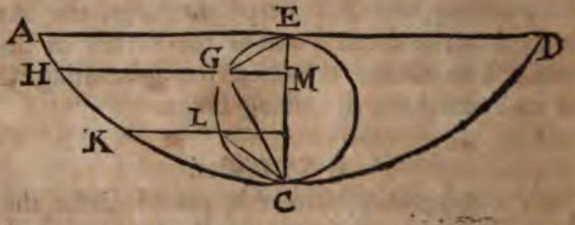
If the generating Circle is placed about the Axis of the Cycloid, and from any Point of the Cycloid C is drawn ordinately to the Axis the right Line CE, meeting with the Circumference of the Circle in D; the right Line CD will be equal to the circular Arch GD, the Arch of the Cycloid GC will be equal to double the Chord GD; and the Semi-cycloid BCG will be equal to twice the Diameter AG: but the right Line CF touching the Cycloid in C, will be parallel to the Chord DG. All theſe things have been demonſtrated by Dr. *Wallis*, and others who have wrote concerning the Cycloid.

THEOR. XLVI.

IN *a Cycloid whoſe Axis is placed perpendicularly to the Plane of the Horizon, and its Vertex downwards, the Times of the Deſcents wherein a moveable Body urged by the Force of its Gravity, and let fall from any Point in it, will arrive at the loweſt Point, are equal amongſt themſelves; and have to the Time of the perpendicular Fall through the Axis of the Cycloid, the Ratio that half the Circumference of a Circle has to its Diameter.*

LET ACD be a Cycloid, whoſe Axis is CE, generating Circle ECG. Since the right Line touching the Cycloid in any Point H, is parallel to the Chord CG drawn in the generating Circle placed on the Axis of the Cycloid; it is manifeſt the moveable Body in its Deſcent will be accelerated with the ſame Force in the Point H, as if it deſcended in the right Line GC; but the Force wherewith it is accelerated in GC, is to the



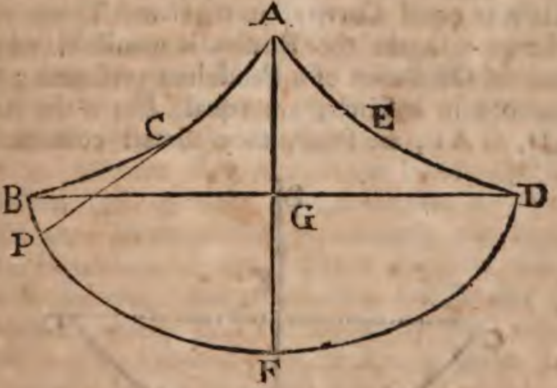


the Force of Gravity as M C to G C: but as M C to G C, ſo is G C to C E, [by Cor. 8. Prop. El. 6.] wherefore the Force wherewith the moveable Body is accelerated in the Point H, is to the Force of Gravity as G C to C E. By the ſame reaſon, the Force of Gravity is to the Force wherewith the moveable Body is accelerated in any other Point K, as C E to C L; wherefore by Equality the Force wherewith the moveable Body is accelerated in H, is to the Force wherewith it is accelerated in K, as G C to L C, or as twice G C to twice L C, that is, as the Curve of the Cycloid H C to the Curve K C. The Forces therefore wherewith the moveable Body in deſcending along the Cycloid is accelerated, are as the Lengths of the Curve run through. Let us now put the right Line *a c* equal to the Length of the Curve A C, and let any moveable Body be ſuppoſed to be urged with the ſame Forces in the right Line *a c* towards *c*, as the moveable Body is urged, in deſcending along the Curve A C; but the Forces wherewith the moveable Body is urged, in any Points H and K of the Cycloid, are as the Lengths H C, K C, or *h c*, *k c*: that is, the Forces in any Places are as the Diſtances of thoſe Places from the Point *c*, and conſequently [by the preceding Theor.] the Times of Deſcent from any Height will be equal. Becauſe therefore in the correſponding Points of the Cycloid and right Line *a c*, the accelerating Forces are equal, the Increments of the Velocities will be alſo equal: as, for example, putting A H $= ab$, the Accelerations in the Points H and *b* will be equal, as alſo in the Points K and *k*, ſo that A K is made $= ak$; and in like manner, in all the other Points of both Lines, which mutually anſwer to one another, the Increments of the Velocities will be equal: ſo that if the Bodies begin to deſcend from correſponding Points, the Sums of the Increments, or the Velocities acquired in deſcribing equal Spaces, will be equal; and conſequently the Times wherein theſe equal Spaces are deſcribed with equal Velocities, will be equal. Therefore the Time of Deſcent from *a* to *c* in the right Line *a c*, is equal to the Time of Deſcent from A to C in the Cycloid; and the Time of Deſcent from *b* to *c* in the right Line *b c*, is equal to the Time of Deſcent from H to C in the Cycloid; and in like manner, the Time of Deſcent through K C, is equal to the Time through *k c*, if the beginning of the Fall is from the Points *k*, K: and ſo of the reſt. But the Time of the Fall from *a* to *c*, is equal to the Time of the Fall from *b* to *c*, or from *k* to *c*; wherefore the Time of Deſcent in the Cycloid from A to C, will be equal to the Time of Deſcent from H to C, or from K to C. The Times therefore of Deſcent, wherein a moveable Body, let fall from any Point in the Cycloid will arrive at the loweſt Point, are equal amongſt themſelves. *Q. E. D.*

MOREOVER, the Time of the Fall from *a* to *c*, is to the Time wherein *a c*, or 2 E C, is paſſed over with the Velocity acquired at the laſt, as the Circumference of a Circle to its Diameter; but the Time wherein 2 E C is paſſed over with the ſame Velocity, is equal to the Time wherein the moveable Body, falling by its Gravity, deſcends through E C, the Axis of the Cycloid: whence the Time of Deſcent through *a c* or A C, will be to the Time wherein the heavy Body deſcends through the Axis of the Cycloid, as the Semi-Circumference of a Circle to its Diameter.

Cor. THE Time wherein a heavy Body deſcends in the Cycloid through the Arch A C, and aſcends through C D, that is, the Time of the Motion in a Cycloid A C D, is to the Time of the perpendicular Fall through the Axis of the Cycloid, as the whole Circumference of a Circle to its Diameter.

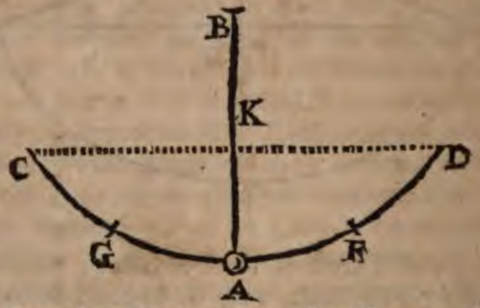
HENCE if the Ball of a Pendulum performs its Vibrations in a Cycloid, whether it deſcribes greater or leſs Arches, each Oſcillation will be always finiſhed in equal Times. Now M. *Huygens*, in the third Part of his *Horologium Oſcillatorium*, hath delivered a Method whereby the Ball of a Pendulum may vibrate in a Cycloid or any other Curve: namely, by finding out a Curve, by whoſe Evolution a given Curve will be deſcribed; then two Plates muſt be bent into the ſame Curvature, betwixt which the Ball being ſuſpended by a String of a determined Length, will deſcribe, not a Circle, but ſome other Curve. Let there be two Plates A C B, A E D, curved into ſimilar and equal Figures; and from the Point A let be ſuſpended the Thred of the Pendulum, which whilſt the Pendulum oſcillates, may be applied to the Plates A C B, A E D, which it conſtantly touches: by the Application of the Thred to the Plates, the Motion of the Pendulum in a Circle is continually prevented, and its Ball is carried thro' the Curve B P F D. The Curve A C B or A E D is called the Evolute, and the Curve B P F D is ſaid to be deſcribed by Evolution. Now if the Curves A C B or A E D are two Semi-



Cycloids, whoſe Axes or Diameters of the generating Circles are equal to F G or A G, namely, to half the Length of the Pendulum; the Curve B P F D through which the Ball deſcends, becomes a whole Cycloid, whoſe Axis F G is half the Length of the Pendulum, as has been demonſtrated by *Huygens* and others.

SINCE the Portion of the Cycloid near the Vertex F is deſcribed by the Motion of a String whoſe Length is A F, and a Circle whoſe Center is A, Semidiameter A F is deſcribed by the ſame String; that Circle paſſing through F, almoſt coincides with the Portion of the Cycloid near the Vertex F, and is of equal Curvity with it. A heavy Body therefore is carried to F, through a ſmall Arch of a Circle, and through the Arch of a Cycloid, to which that Circle is of equal Curvity, in the ſame Time.

HENCE again the Reaſon is manifeſt, why the Times of Oſcillation of a Pendulum performing ſmall Vibrations in a Circle, are equal: For if the Arches C A D, G A F, are ſmall, they almoſt coincide with the Portion of a Cycloid near its Vertex; the Axis of the Cycloid being A K, half the Length of the Pendulum. So that the heavy Body deſcends through the Arches of a Circle C A or G A, almoſt in the ſame Time, as it would deſcend through the Arches of a Cycloid nearly coinciding with them: but it deſcends in equal Times through any Arches of a Cycloid;



cloid; wherefore the heavy Body alſo will fall in equal Times through the ſmall circular Arches C A, G A; and conſequently the whole Oſcillations through the Arches C A D, G A F, are performed in equal Times.

The Time therefore wherein a Pendulum performs a very ſmall Oſcillation in a Circle, is equal to the Time wherein an Oſcillation is performed through the Arch of a Cycloid, whoſe Axis is half the Length of the Pendulum. But the Time wherein an Oſcillation is performed in a Cycloid, is to the Time of the perpendicular Fall through the Axis of that Cycloid, that is, through half the Length of the Pendulum, as the Circumference of a Circle to its Diameter. And it hence follows, that the Time of any the ſmalleſt Oſcillation is to the Time of the Fall through the Length of the Pendulum, in a conſtant Ratio; namely that, which the Circumference of a Circle has to its Diameter multiplied by the ſquare Root of the Number 2.

If in different Regions of the Earth, the ſame Pendulum ſhall perform its Oſcillations in unequal Times, the Times of Deſcent through the Length of the Pendulum in the different Regions will be likewiſe unequal; and where the Oſcillations proceed ſlower, there alſo a heavy Body will deſcend ſlower in the Perpendicular, and in a given Time will deſcribe by falling a leſs Space. And it is certain by Experiments, that in Places ſituated near the Equinoctial, the Oſcillations of a Pendulum of the ſame Length are performed in a longer Time, than in Places that are more remote from the Equinoctial: ſo that heavy Bodies in thoſe Regions deſcribe in a given Time by falling a leſs Space, and are accelerated in their Motion by a leſs Force than in our Regions, which have a greater Latitude: ſo that it is confirm'd by Experiments, that the Action of Gravity is leſs in ſuch Places as have leſs Latitude, than in thoſe that are nearer the Pole.

THIS Decreaſe of Gravity ariſes from the centrifugal Force: For ſince from the Rotation of the Earth about its Axis, every Body endeavours to recede from the Center of the Circle it deſcribes, by how much the greater thoſe Circles are, by ſo much the greater is the centrifugal Force of the Bodies deſcribing theſe Circles; which Force therefore is always as the Sine of the Diſtance of the Place from the Pole, which as it is greateſt under the Equinoctial, is nothing at all under the Pole itſelf: ſo that the Force of Gravity is the leaſt under the Equinoctial, but at the Pole the greateſt.

BEFORE we finiſh this matter, we ſhall here give a Solution of a famous Problem, firſt ſought out by *Gallileo*, after propoſed by *John Bernouilli* to the Geometers, namely, in the beginning of the Year 1696; and ſolved by the famous Geometers Sir *Iſaac Newton*, Mr. *Leibnitz*, *James Bernouilli*, the Marquiſs *de l'Hoſpital*, and others. The Problem itſelf was thus propoſed.

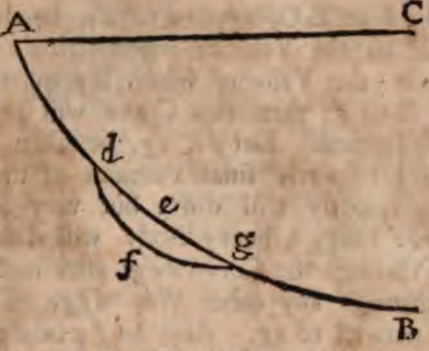
GIVEN *in a vertical Plane two Points* A *and* B, *to aſſign the Way of a moveable Body, through which it deſcending by its Gravity, and beginning its Motion at the Point* A, *it may arrive in the ſhorteſt Time poſſible at the other Point* B.

THE above-mentioned Geometers found out, that this Line would be a Cycloid paſſing through the Points A, B, whoſe Baſe lay in the Horizontal Line drawn through A: to demonſtrate which, we ſhall premiſe the following

LEMMA.

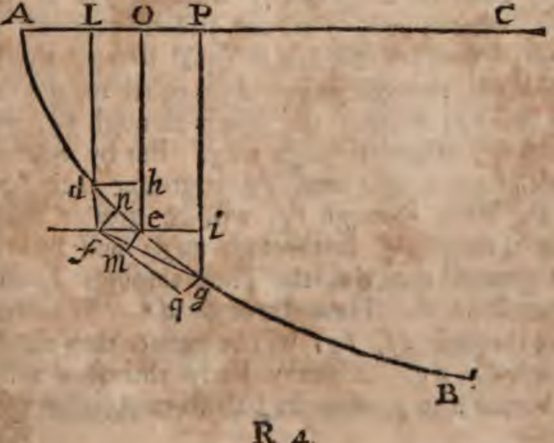
IF A d g B *is the Line of ſwifteſt Deſcent, a heavy Body will deſcend quicker from any Point of it* d, *to any other Point of it* g, *after the Fall from* A, *through the Curve itſelf* d e g, *than through any other way whatever.*

FOR



FOR if you ſay the heavy Body may deſcend quicker through *dfg*, therefore the Way A *dfg* B will be paſſed over in a ſhorter Time than A *deg*; and conſequently the Curve A *deg* B will not be the Curve of ſwifteſt Deſcent: which is contrary to the Hypotheſis.

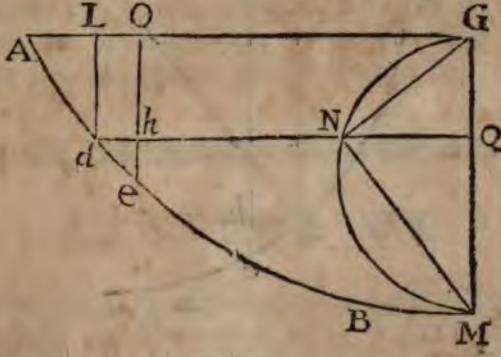
NOW let A *deg* be the Curve, its Axis A C, and Ordinate applicate *d* L; the Fluxion or momentaneous Increment of the Axis, let be L O $= dh$; but the Fluxion of the Curve let be *de*: and let the Rectangle under a given right Line, which we will



call a and db or LO, applied to de, be always proportionable to the Velocity wherewith de is paſſed over, that is, the Velocity which is acquired by falling from A to d; then this Curve will be the Line of ſwifteſt Deſcent. Let de, eg, be taken two contiguous and infinitely ſmall Portions of the Curve; which conſequently will differ but very little from right Lines: I ſay, a heavy Body will deſcend in a leſs Time through the Curve deg, after its Fall from A, than through any other Way dfg. Through f draw fq parallel to eg. And let fq be ſuppoſed to be paſſed over with the ſame Celerity as eg is paſſed over; and let fn be perpendicular to de, as alſo me, gq to fq. And by reaſon the Triangles fne, deh, are equi-angular, as likewiſe fme, gei; de is to db as fe to ne: ſo that it will be $ne=\frac{db\times fe}{de}$; alſo by reaſon ge is to ei as fe to fm, it will be $fm=\frac{ei\times fe}{ge}$, but it is $\frac{db\times fe}{de}:\frac{ei\times fe}{ge}::\frac{db}{de}:\frac{ei}{ge}::\frac{db\times a}{de}\ \frac{ei\times a}{ge}$; that is, ne is to fm, as the Velocity whereby ne is paſſed over to the Velocity whereby fm is paſſed over: whence ne, fm, are paſſed over in equal Times. And becauſe mq is equal to eg, the Time of Deſcent through mq will be equal to the Time through eg; ſo that the Time through fq, will be equal to the Time through neg. But by reaſon the Angle at q is a right one, fg is greater than fq; ſo that the Time through fg, will be greater than the Time through fq, or through neg: and by reaſon df is greater than dn, the Time through df will be greater than the Time through dn. Whence the Time through df, fg, will be greater than the Time through dn, ng. A heavy Body therefore will deſcend from d to g, after its Fall from A, through the

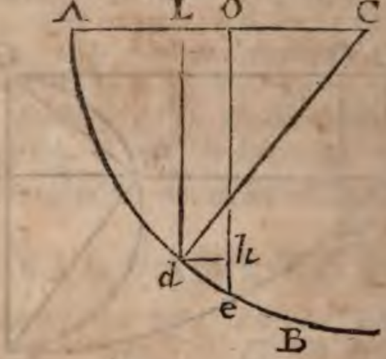
Curve *deg*, in a leſs Time than through any other Way; and conſequently the Curve A *deg* B will be the Way of ſwifteſt Deſcent.

LET ABM be a Cycloid, paſſing through B, whoſe Baſe let be a Horizontal right Line drawn through A; that will be the Line, on which a heavy Body deſcending, will proceed from A to B, in the leaſt Time poſſible. Let GNM be half the generating Circle, whoſe Diameter GM let be called *a*, and let *de* be any infinite ſmall Part of the Cycloidal Curve, which will differ but inſenſibly from its Tangent in *d*; ſo that it will be parallel to the right Line NM: whence the Triangles *dhe*, NQM, GMN, will be equiangular. Wherefore *de* is to *dh*, as GM or *a* to GN; and conſequently $dh \times a = de \times GN$; and $\frac{dh \times a}{de} = GN$. But [by Cor. of Theor. 43.] GN is as the Velocity acquired by the heavy Body in falling from the Height GQ, or L*d*; that is, as the Velocity wherewith the infinitely ſmall Line *de* is paſſed over. Wherefore $\frac{dh \times a}{de}$ will be proportionable to the Velocity wherewith *de* is paſſed



paſſed over. The Cycloid A *d e* B therefore is the Line of ſwifteſt Deſcent. *Q. E. D.*

If the Velocity is ſuppoſed to be as the Height from whence the heavy Body falls, the Line of ſwifteſt Deſcent will be the Portion of the Circumference of a Circle, whoſe Center is in the horizontal Line drawn through A. For by reaſon the Triangles *d h e*, *d* L C, are equi-angular, *d h* is to *d e* as *d* L to *d* C; and conſequently, it will be $dh \times dC = de \times dL$, and $\frac{dh \times dC}{de} = dL$. But by Hypotheſis *d* L is proportionable to the Velocity: wherefore if *d* C is called *a*, $\frac{dh \times a}{de}$ will be proportionable to the Velocity. Therefore in this Hypotheſis the Portion of the Circumference A *d e* B will be the Way of ſwifteſt Deſcent.



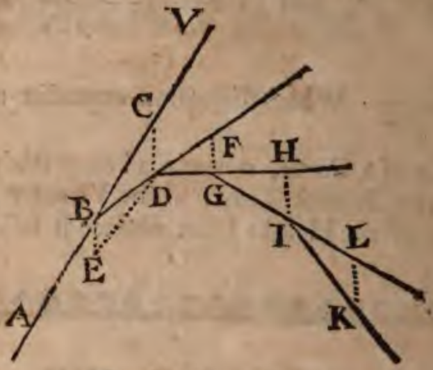
If the Velocity in any Point be as the Dignity *m* of the Height whence the Body fell, and let A L be called x, *d* L be y; it will be $dh = \dot{x}$, $he = \dot{y}$, and $de = \sqrt{\dot{x}^2 + \dot{y}^2}$. Wherefore from the Nature of the

Curve, it will be $\frac{a^{m}\dot{x}}{\sqrt{\dot{x}^{2}+\dot{y}^{2}}}=y^{m}$; whence $\frac{a^{m}\dot{x}^{2}}{\dot{x}^{2}+\dot{y}^{2}}$ $=y^{2m}$, and $a^{2m}\dot{x}^{2}=y^{2m}\dot{x}^{2}+y^{2m}\dot{y}^{2}$, and $a^{2m}\dot{x}^{2}-y^{m2}\dot{x}^{2}=y^{2m}\dot{y}^{2}$, and $\dot{x}^{2}=\frac{y^{2m}\dot{y}^{2}}{a^{m2}-y^{2m}}$, and $\dot{x}=\frac{y^{m}\dot{y}}{\sqrt{a^{2m}-y^{2m}}}$. Which Equation expreſſes univerſally the Nature of a Curve, wherein a heavy Body will deſcend in the ſhorteſt Time, if the Velocity is as any Dignity *m* of the Height from whence it fell.

LECTURE XVI.

WE have, in the foregoing Lecture, explained the Motions of heavy Bodies on inclined Planes, or curve Superficies; and their principal Properties, as far as was conſiſtent with our intended Brevity. It now remains that we ſpeak of the Phenomena of Projectiles: and firſt of all, we ought to find out the Nature of that Line, which a moveable Body projected in free and unreſiſting Spaces, will deſcribe by the Force of its Gravity. And indeed if the heavy Body is projected directly upwards or downwards, it will be moved in a right Line; and it is manifeſt from the former Lectures, that its Motion will be an uniformly retarded or accelerated Motion, as it is projected either upwards or downwards. But if is is projected in a horizontal Direction, or in any other that is oblique to the Horizon, it will be carried in a curve Line.

For let the Body be projected or caſt from A, in the Direction A V; by the firſt Law of Nature, if no other Force acted on it, it would proceed always in the ſame right Line, with the ſame Velocity: ſo that it would deſcribe equalSpaces, A B, B C, in equal Times. Let us therefore diſtinguiſh the Time into equal Particles; and if after the firſt Particle of Time, when the moveable Body ſhall arrive at B, ſome Force is ſuppoſed to act on it, by one Impulſe only; and to communicate to it a Motion, whereby it would be carried (the other Motion being taken away) in a Direction perpendicular to the Horizon, through the right Line B E, in the Time that it would deſcribe the right Line B C; compleat the Parallelogram C B E D: it is manifeſt from Cor. 2. Theor. 30. that the Body, by the Motion compounded of both the others, will be moved through the Diagonal B D, and afterwards the projected Body would always proceed in this right Line, if no new Force turned it out of its Path; and it would in an equal Time deſcribe a Space D F equal to B D. But if in the Point D, the ſame Force ſhould act again with a like Impulſe, whereby the Body would be

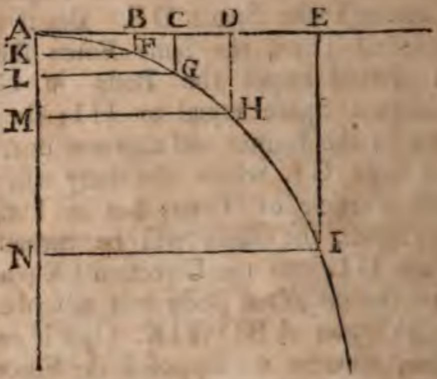


be carried downwards in that Time through a Space equal to FG: the Motion of the Body, compounded of both Motions, would be through the right Line DG, which the Body will deſcribe in the ſame Time, as it would, without this new Impulſe, have proceeded through the Space DF. But if after the third Particle of Time, the ſame Force ſhould act again, and ſhould impel the Body in G downwards, through a Space equal to HI; the Motion compounded of the former and this new one, will be in the right Line GI, which the Body will deſcribe in the fourth Particle of Time: but in I the ſame Force acting again, the Body will be turned out of its right Path GL into the Direction IK; and after this manner the projected Body will deſcribe by its Motion the Polygon ABDGIK. But if each Particle of Time, wherein we ſuppoſed the Force to act, is diminiſhed *in infinitum*, and their Number is augmented: the Sides of the Polygon will be diminiſhed *in infinitum*, and their Number will be increaſed *in infinitum*, and conſequently the Polygon will be turned into a Curve: that is, if the Force impelling downwards is ſuch, as, that it ſhould act always and without ceaſing, of which nature is the Force of Gravity, the projected Body, this Force acting, will be carried along a Curve.

THEOR. XLVII.

A PROJECTED *Body, whoſe Line of Direction is parallel to the Plane of the Horizon, deſcribes by its Motion a Parabola.*

IF the heavy Body is thrown by any extrinſecal Force, as that of a Gun, or the like, from the Point A, ſo that the Direction of its Projection is the Horizontal Line AD; I ſay, the Path of this heavy Body will be a Semi-Parabola. For if the Air did not reſiſt it, nor was it acted on by its Gravity, the Projectile would proceed with an equable Motion, always in the ſame Direction; and the Times wherein the Parts of Space A B, A C, A D, A E, were paſſed over, would be as the Spaces A B, A C, A D, A E, reſpectively. Now if the Force of Gravity is ſuppoſed to take place, and to act in the ſame Tenour, as if the heavy Body were not impelled by any extrinſecal Force; that Body would conſtantly decline from the right Line A E, and the Spaces of Deſcent or the Deviations from the Horizontal Line A E, will be the ſame as if it had fallen perpendicularly. Wherefore if the Body falling perpendicularly by the Force of its Gravity, paſſed over the Space A K in the Time A B, deſcended through A L in the Time A C, and through A M in the Time A D; the Spaces A K, A L, A M, will be as the Squares of the Times, that is, as the Squares of the right Lines A B, A C, A D, or of K F, L G, M H. But ſince the Impetus in the Direction parallel to the Horizon always remains the ſame; for the Force of Gravity, that only ſollicites the Body downwards, is not in the leaſt contrary to it: the Body will be equally promoted

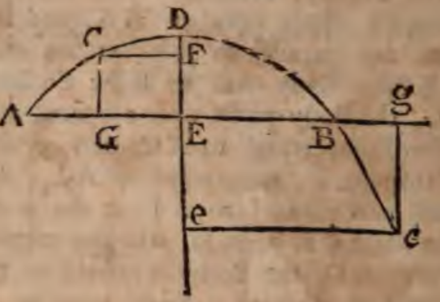


promoted forwards in the Direction parallel to the Plane of the Horizon, as if there was no Gravity at all. Wherefore ſince in the Time A B, the Body paſſes over a Space equal to A B; but being compelled by the Force of Gravity, it declines from the right Line A B through a Space equal to A K, and B F being made equal and parallel to A K, at the end of the Time A B, the Body will be in F: So ſince in the Time A C, the Body paſſes over a Space, in the Horizontal Direction, equal to A C, and in that Time deſcends through a Space equal to A L; if C G is made equal and parallel to A L at the end of that Time, the Body will be in G. In like manner, ſince in the Time A D, the Body is carried in the Horizontal Direction through a Space equal to A D, by the Action of Gravity, it would in the mean while deſcend through a Space equal to A M; and D H being put equal to A M, at the end of the Time A D, the Body will be in H. And the Path of the Projectile will be in the Curve A F G H; but becauſe the Squares of the right Lines K F, L G, M H, are proportionable to the Abſciſſas A K, A L, A M, the Curve A F G H will be a Semi-Parabola. The Path therefore of a heavy Body projected according to the Direction A E, will be a ſemi-parabolical Curve. *Q. E. D.*

LEMMA.

LET A D B *be ſuch a Curve, as that the Perpendicular* C G *being let fall from any of its Points* C *to* A B, *the Rectangle under* A G, G B, *is equal to the Rectangle under* C G, *and a given right Line* L; *that Curve will be a Parabola.*

LET A B be biſected in E, and the Perpendicular D E be erected; by Hypotheſis, the Rectangle under D E and L, will be equal to the Rectangle under AE, EB, or $AEq. =$ (by 5 *El.* 2.) $AG \times GB + GEq. = CG \times L + GEq. = EF \times L + CFq.$ Wherefore the Rectangle under DF and L will be equal to the Square of CF, which is the



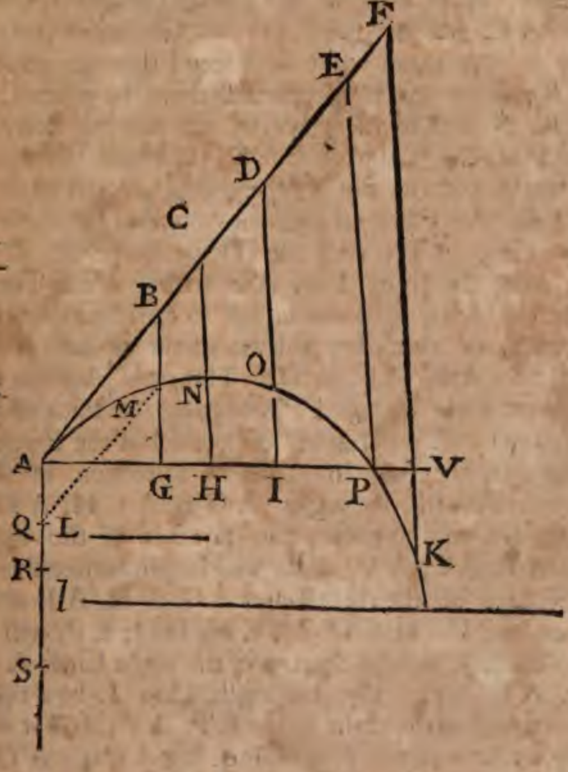
Property of the Parabola. If the Point *g* falls in AB produced, which happens when the Curve deſcends below AB, the ſame Parabola will be the *Locus* of the Point *c*; for (by 6 *El.* 2.) it is $Egq. = (ecq. =) Ag \times gB + EBq. = L \times cg + L \times DE = L \times De$: which is the Property of the Parabola.

Cor. THE right Line L is the *Latus Rectum*, or *Parameter* of the Parabola.

THEOR. XLVIII.

THE *curve Line, that is deſcribed by a heavy Body projected obliquely and upwards, according to any Direction, is a Parabola.*

LET AF be the Direction of Projection, any ways inclined to the Horizon. Gravity being ſuppoſed not to act, the moving Body would always continue its Motion in the ſame right Line, by the firſt Law of Nature, and would deſcribe the Spaces AB, AC, AD, proportionable to the Times. But by the Action of Gravity it is compelled continually to decline from the Path A F, and to move in a Curve; I ſay, this Curve will be a Parabola. Let us ſuppoſe the heavy Body falling perpendicularly in the Time A B, through the Space A Q, but in the Time A C,



through the Space A R; and in the Time A D, thro' the Space A S; the Spaces A Q, A R, A S, will be as the Squares of the Times, or as the Squares of the right Lines A B, A C, A D. But becauſe the moving Body not being acted upon by Gravity, in the Time A B would paſs over the Space A B, and by the Action of Gravity would in the mean while deſcend through a Space equal to AQ, it is manifeſt, if in the Perpendicular BG, there is taken $BM = AQ$, the Place of the heavy Body, at the end of the Time AB, would be M. In like manner, ſince the moving Body, by the Impetus firſt impreſſed, in a Time as AC, ought to paſs over the Space AC, but by the Force of Gravity it is compelled in the mean while to deſcend through a Space $= AR$; if in the Perpendicular there be taken $CN = AR$, N will be the place of the moving Body at the end of the Time AC. So likewiſe the Space DO being put, in the Perpendicular, equal to AS, O will be the Place of the moving Body at the end of the Time AD; and the Deviations BM, CN, DO, from the right Line AF ariſing in the Times AB, AC, AD, will be equal to the Spaces AQ, AR, AS, ſo that they will be as the Squares of the right Lines AB, AC, AD. Through A draw the horizontal right Line AP, meeting the Path of the Projectile in P. From P let be erected the Perpendicular PE, meeting the Line of Direction in E; and by reaſon the Triangles AGB, ACH, ADI, AEP, are equi-angular; the Squares of the right Lines AB, AC, AD, AE, will be proportionable to the Squares of the right Lines AG, AH, AI, AP: ſo that the Deviations BM, CN, DO, EP, will be proportionable to the Squares of the right Lines AG, AH, AI, AP. Let the right Line L be a third Proportional to the right Lines EP, AP; and it will be [by 17 *El.* 6.] $L \times EP = APq.$ but $APq. : AGq. :: EP : BM :: L \times EP : L \times BM$; whence ſince it is $L \times EP = APq.$ it will be $L \times BM = AGq.$ In like manner, it will be $L \times CN = AHq.$ and $L \times DO = AIq.$ But becauſe it is $BG : AG ::$ ($EP : AP ::$ by Hypotheſis) $AP : L$; it will be $L \times BG = AG \times AP = AG \times AG + AG \times GP = AGq. + AG \times GP$. But it has been ſhewn, that it is L×BM=AGq. wherefore it will be L×BG—L×BM=AG×GP, that is, L×MG =AG×GP. By the ſame way of Reaſoning it will be L×NH=AH×HP, and L×OI= AI×IP, as alſo L×VK=AV×VP. Wherefore by the preceding Lemma, the Curve AMNOPK, wherein the Projectile is moved, will be a Parabola. *Q. E. D.*

Cor. 1. The right Line L is the *Latus Rectum* or Parameter of the Parabola, that belongs to its Axis.

Cor. 2. Let be AH=HP, and it will be L×CN =AHq.=L×NH, whence it will be NH=CN; and conſequently the right Line AF being the Line of Direction of the Projectile, will [by Prop. 33. Lib. 1. of *Apollonius*] touch the Parabola.

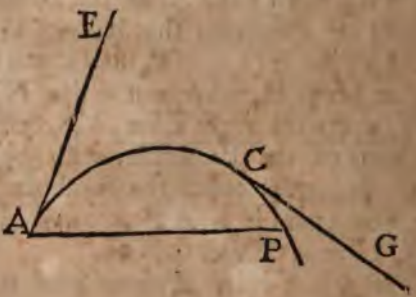
Cor. 3. Because it is AP=2AH; it will be PE=2CH=4CN or 4NH.

Cor. 4. If *l* is a third Proportional to the right Lines PE, AE, *l* will be the *Latus Rectum*, or Parameter of the Parabola, that belongs to the Diameter AS. For becauſe PE, AE, *l*, are continual Proportionals; it will be *l*×PE=AEq. but it is AEq. : ABq. or QMq. : : PE : BM or AQ : : *l*×PE : *l*×AQ. Wherefore ſince it is AEq.×PE, it will be QMq.=*l*×AQ: wherefore *l* will be the Parameter belonging to the Diameter AS.

Cor. 5. But it is *l*=PE+L=4NH+L= four times the Height of the Parabola +L. For it is *l*×PE=AEq.=APq.+PEq.=L×PE+ PEq.=$\overline{\mathrm{L}+\mathrm{PE}}$×PE. Wherefore it will be *l*=L+PE=L+4NH.

Cor. 6. If the Times AB, BC, CD, are made equal; the Horizontal Spaces AG, GH, HI, will be equal; that is, if a heavy Body, in its Motion, deſcribes a Parabola, in equal Times it will advance equally according to a Direction that is parallel to the Horizon; and in every Point of the Parabola, the Horizontal Impetus will remain the ſame as it was at the beginning of Motion.

Cor. 7. IF a Body projected from A in the Direction A E, deſcribes the Parabola A C P; in any Point C it will endeavour, by the firſt Law of Nature, to run out along the Tangent C G, with all the Velocity that it has in the Point C, and is ſolely retained in the Parabolical Curve by the Force of Gravity. But if another heavy Body ſhould be projected from C in the Direction C G, with the Velocity that the Body projected from A, has in the ſame Point C; that other Body will deſcribe the ſame Parabola C P. For in the Point C, the Direction, Velocity, and Force of Gravity of both Bodies are the ſame; wherefore the Way of both will be the ſame.



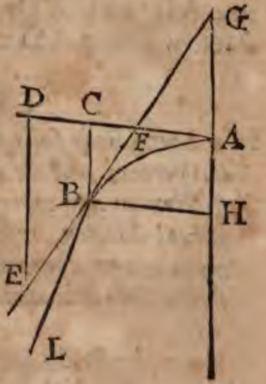
Cor. 8. HENCE if a heavy Body is projected downwards, in a Direction oblique to the Horizon; the Path of the Projectile will be a Parabola.

THEOR. XLIX.

THE *Impetus of a projected Body in different Points of the Parabola, are as the Portions of the Tangents intercepted betwixt two right Lines parallel to the Axis.*

LET a heavy Body deſcribe the Parabola A B L, which let the right Lines A D, B E, touch in the

Points A and B. The Impetus of the Body in the Points A and B, will be as C D, E B, the Portions of the Tangents intercepted betwixt two right Lines parallel to the Axis. For if the Body ſhould be deprived of its Gravity in the Point A, it would run out in the Tangent A C, with the ſame Impetus that it has in the Point A. So likewiſe the moving Body in B having loſt its Gravity, would proceed in the Tangent BE with all the Velocity that it has in the Point B. But in the Points A and B, the Horizontal Impetus remains the ſame, as appears by Cor. 6. of the preceding Theor. ſo that the Body in A going out into the Tangent A D, and in B into the Tangent B E, in equal Times it will proceed through equal Spaces according to the Horizontal Direction. Therefore C D in the Tangent A D, and B E in the Tangent B E, will be paſſed over in equal Times; but the Velocities, or Impetus of the moving Bodies, are as the Spaces paſſed over in equal Times: wherefore the Impetus of the moving Body in A, is to the Impetus of the ſame Body in B, as C D to B E. *Q. E. D.*



Cor. If A is the Vertex of the Parabola, and the Tangent produced, till it meets the Axis in G; the Impetus in A will be to the Impetus in B, as the Ordinate B H to the Tangent B G: for it is C D : B E : : C F : B F (by reaſon the Triangles C B F, B H G, are ſimilar) : : B H : B G.

Defin. Let A C F be a Parabola, in whoſe Axis produced beyond the Vertex let be taken G A = $\frac{1}{2}$ the Parameter.

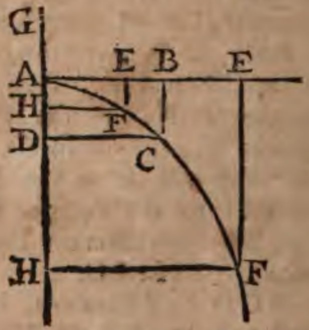
See the Fig. of the following Theorem.

The Line GA is called the *Sublimity* of the Parabola, and if beneath the Vertex be taken $AD = AG$, and DC be an Ordinate to the Axis, is will be $DC = 2\,AD$ or $2\,AG$; for from the Nature of the Parabola, the Rectangle under the *Latus Rectum* $= 4\,AD$ and AD, that is, $4\,ADq. = DCq.$ ſo that it will be $2\,AD = DC$.

THEOR. L.

If a heavy Body falls from the Sublimity of a Parabola to its Vertex, and the Motion acquired in falling, by Reflection or any other way ſhould be changed into a Horizontal Motion, inſomuch that the Body may begin to move a-new downwards; the projected Body will deſcribe that Parabola.

Let the Body fall from the Point G, the Sublimity of the Parabola ACF; and in A by Reflection, or any other way, let the Motion acquired in falling be changed into the Horizontal one in ABE; or, which is the ſame thing, let the Body be projected in the Direction AE, with the Velocity that is acquired by falling through GA; I ſay, that heavy Body will deſcribe in its Motion the Parabola ACF. Let be $AD = AG$, and it will be $DC = 2\,AG$. Draw



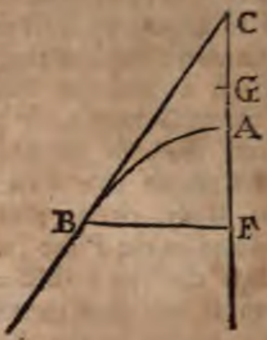
Draw C B parallel to A D; and from any Point F of the Parabola, draw F H parallel to A E, and F E to H A: If there was no Gravity, and the Body was projected in the Direction A E with the Velocity that is acquired in falling from G into A, it would be carried in the ſame Time through the Double of G A; ſo that it would deſcribe in that Time A B = D C = 2 G A. But the Body, on account of its Gravity, beginning to deſcend in the Point A, will fall in the ſame Time through the Space B C = A G; wherefore in its Motion it will paſs through the Point C in the Parabola. Again, let the Body be ſuppoſed by its Horizontal Motion (abſtracting from that which ariſes from its Gravity) to arrive in a certain Time at E, beyond or on this ſide B; and ſince the Motion in the Direction parallel to the Horizon remains equable, A B, A E, will be as the Times wherein they are paſſed over. But the Deſcents or Deviations of the moving Body from the right Line A E, are as the Squares of the Times wherein they are made: wherefore by reaſon B C, E F are proportionable to the Squares of the right Lines A B, A E; ſince C is the place of the heavy Body at the end of the Time A B, F will be the Place of the ſame Body at the end of the Time A E; and ſo the heavy Body will be always found in the Parabola A C F. *Q. E. D.*

Cor. HENCE the Velocity of a heavy Body in the Vertex of the Parabola which it deſcribes, is that which is acquired by falling from the Sublimity of that Parabola.

LEMMA.

LET B A *be a Parabola, whoſe Axis is* A F, *Sublimity* A G, *and Tangent* B C, *Ordinate* B F; *it will be* B F q. : B C q. : : G A : G F.

FOR it is [by 33 Lib. 1, of *Apollonius*'s Conicks] C F = 2 A F, and from the Nature of the Parabola



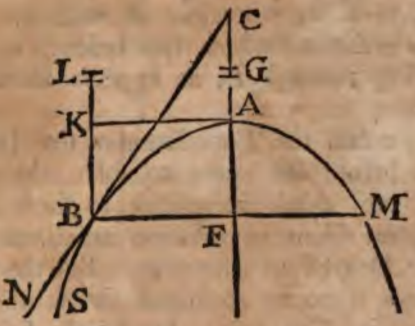
$4GA \times AF = BFq.$ Wherefore it will be $BFq. : BCq. :: 4GA \times AF : 4GA \times AF + CFq :: 4GA \times AF : 4GA \times AF + 4AFq. :: GA : GA + AF$ or GF. *Q. E. D.*

THEOR. LI.

A Heavy *Body being projected directly upwards, with the ſame Impetus wherewith another heavy Body is projected obliquely, will aſcend to a Height equal to the Height and Sublimity, taken together, of the Parabola, which the oblique Projectile will deſcribe in its Motion.*

Let from B be projected a heavy Body in the Direction BC, deſcribing in its Motion the Parabola BAM, whoſe Axis is AF, Vertex A, and Sublimity GA: I ſay, if the ſame or another heavy Body be projected from B directly upwards, with the ſame Impetus, it will aſcend to L; ſo that BL will be equal to FG, the Height and Sublimity of the Parabola taken together. By Cor. of Theor. 49. the Impetus of the heavy Body in B, is to the Impetus of the ſame Body in A, as BC to BF; but the Impetus acquired by falling from G to F, is to the Impetus acquired by falling from G to A, in a ſubduplicate Ratio of GF to GA, that is, (by reaſon it is

BCq. : BFq. : : GF : GA) as BC to BF. Wherefore the Impetus in B, will be to the Impetus in A, as the Impetus acquired by falling from G to F, to the Impetus acquired by falling from G to A: but the Impetus of the heavy Body in the Vertex A, is that which is acquired by falling from G to A; wherefore the Impetus or Velocity of the ſame Body in B, is that which is acquired by falling from G to F, or from L to B, which Height is equal to the Height and Sublimity of the Parabola taken together. But the heavy Body projected directly upwards with the ſame Impetus, will aſcend to L; wherefore if a heavy Body is projected directly upwards with the Impetus that another heavy Body, deſcribing the Parabola B A M, has in the ſame Point B, it will aſcend to a Height equal to the Height and Sublimity of the Parabola taken together. *Q. E. D.*

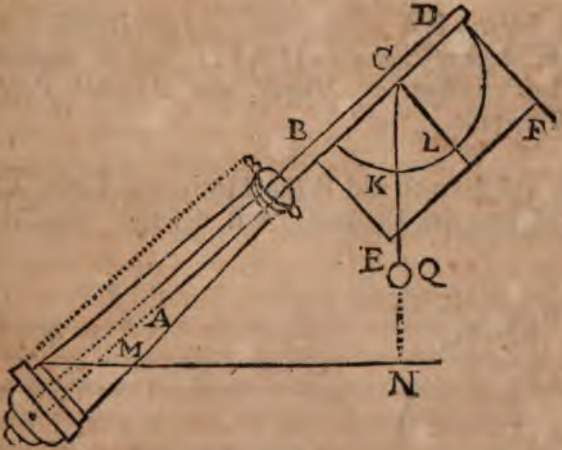


Cor. 1. If a heavy Body falls from L to B, and the Impetus acquired in the Fall remaining the ſame, the Direction of Motion ſhould be changed, by Reflection, or the like manner, into the right Line B C, or B N, ſo that the heavy Body begins to deſcend again; it will in its Motion deſcribe the Parabola S B A M.

Cor. 2. The Impetus in any Point B of a Parabola, is that which is acquired, by falling through the fourth part of the *Latus Rectum* belonging to the Diameter that paſſes through that Point. For it is $LB = \frac{1}{4}L + KB$; wherefore it will be $4LB = L + 4KB =$ *Latus Rectum* that belongs to the Diameter paſſing through B, as appears from Cor. 5. Theor. 48.

Having laid the Foundation of the Doctrine of Projectiles, before we come to ſolve the following Problems, we think it proper to ſhew the way, whereby great Guns or Cannon are directed according to any Degree of Elevation. But the Direction of a Cannon is to be reckoned the ſame with the Direction of its Concavity: for by the firing of the Gun-Powder, the Bullet is emitted according to the Concavity of the Cannon or Mortar; and if it was not for Gravity, it would proceed in that right Line produced: ſo that that right Line is the Direction of the Ordnance.

Wherefore that the Ordnance may be aimed at the Mark, it ought not to be directed according to its outward Metal, ſince Ordnance are thicker towards their Breech than near their Orifice, becauſe their Reſiſtance ought to be greateſt in that part, which ſuffers moſt from the Gun-powder: whence, that a Cannon may be the more readily directed, ſomething is added to the Orifice, (which they call *Diſpart*) that its Thickneſs may equal the Thickneſs of its Breech, and then it is levelled by a Line parallel to the Concavity of the Piece; and after the abovemention'd manner the Ordnance is aimed directly at the Mark. When Walls are to be battered, or any thing to be done, where a great Impetus is required, and the Mark is not above 200 Paces off, and the Engine ſufficiently large; in ſuch Shots, beſides what has been ſaid, and the Experience of making proper Charges of Powder, and ſuitable Balls, there is no other Artifice required. But ſince moſt commonly the Citadels, or Enemy that is to be fired upon, by reaſon of the too great diſtance, cannot be hit by a direct Level; or when the Houſes of the City are to be broken down or burnt by the flinging of Bombs; the Engine muſt be elevated in an Angle inclined to the Horizon: to which end, there is uſed a Ruler A B C D, whereunto is fixed a Parallelogram B E F D, on which is inſcribed a Semi-circle divided into Degrees, from whoſe Center hangs a Plummet; but the Extremity of the Ruler A muſt be inſerted into the mouth of the Engine, and be held in a Situation that is parallel to its Axis. And ſo the Ordnance is to be raiſed or depreſſed, till the Perpendicular C Q touches, in the Limb of the Semi-circle, the Point K, namely, the Degree of the Elevation required, reckoning from L towards B; for it is manifeſt, that the Angle L C K is equal to the Angle C M N, the Elevation of the Engine, becauſe the Angle M C N is the Com-



Complement of both to a right Angle. The Parallelogram B E F D is frequently uſed alone without the Ruler, by applying its Side B E to the Mouth of the Engine, whereby the Perpendicular C Q will ſhew the Degree of Elevation

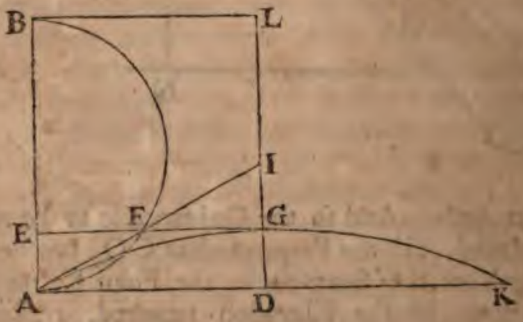
See the Fig. of the next Problem.

Defin. By the Impetus deſigned by any Perpendicular A B, we underſtand an Impetus ſufficient to hurl the propoſed heavy Body from A to the higheſt Point B of the Perpendicular A B; or, which is the ſame thing, the Impetus acquired by falling from B to A; for by no other way can an Impetus be reduced under any certain and univerſal Rule, than by determining it after this manner by Spaces.

PROBL. VIII.

THE *Impetus* B A *being given, and the Direction* A I, *or the Angle of Elevation* D A I; *to find the Amplitude of the Projection, its Height, and the whole Path it will deſcribe.*

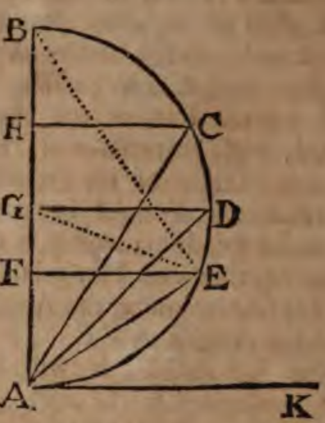
DRAW from A and B the Horizontal Lines A D, B L. Upon the Diameter A B deſcribe the Semi-



circle A F B, which will cut the Line of Direction A I in F: through F draw E F parallel to the Horizon,

zon, and produce it to G, ſo that it may be $GF = EF$; and alſo through G draw perpendicular to the Horizon the Line LGD; and with the Vertex G deſcribe the Parabola AGK paſſing through A: I ſay, this will be the Path of the Projectile, whoſe Direction is AI, and Impetus AB; ſo that DG or AE will be the Height of the Projection. Double AD, or quadruple EF, will be its Amplitude or whole Horizontal Random; and BE or LG will be the Sublimity of the ſame Parabola. In the Triangles AEF, IGF, by reaſon the Angles at E and G are right, and the Angles AFE, GFI, at the Vertex equal, alſo $EF = GF$; it will be $IG = AE = DG$, and conſequently the right Line AI will touch the Parabola. And becauſe it is $AD = EG = 2\,EF$; it will be $AD\text{q.} = 4\,EF\text{q.} = 4\,BE \times EA = 4\,LG \times GD$ = Rectangle under the *Latus Rectum* and GD: wherefore it will be $4\,LG$ = the *Latus Rectum* of the Parabola, whence LG will be the Sublimity of the ſame Parabola. Wherefore [by Cor. Theor. 51.] if the heavy Body falls from B to A, and is projected in the Direction AI with the Impetus acquired in falling, it will deſcribe the Parabola AGK.

Cor. 1. HENCE it is manifeſt, from the Impetus AB of any Engine being given, about which is deſcribed the Semi-circle ADB, that the Heights and Amplitudes of all Projectiles, which can be made by the ſame Engine, are given. For example, the Impetus AB remaining always the ſame, the Projection made in the Direction AE, has its Height AF, and its Amplitude the Quadruple of EF: in like manner, it being made in the Direction AD, its Height will be AG, and its Amplitude the Quadruple of GD; and ſo of the reſt. Whence if the Angle of Elevation DAK is half a right one, the Quadruple of GD will be the greateſt of all the Amplitudes, which can



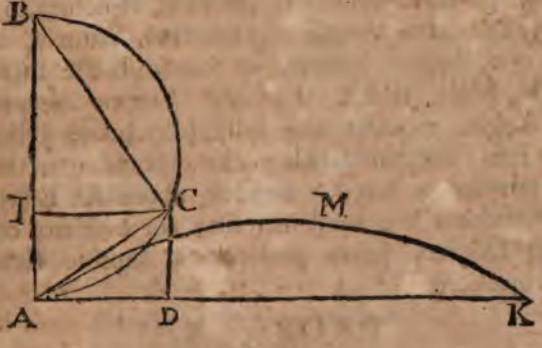
can be made with the ſame Impetus; and the Amplitudes of Projections, which are equally diſtant, from the Projection whoſe Elevation is half a right Angle, are equal: as, for example, the Projection in the right Lines A E, A C, (the Angles D A E, D A C, being made equal) have their Amplitudes quadruple of E F, and quadruple of H C, which are equal. Beſides, the Amplitude of the Projection whoſe Elevation is half a right Angle, is $= 4\,GD = 4\,GB =$ *Latus Rectum* of the Parabola. But the perpendicular Projection upwards, that is, the Impetus of the Projection, will be equal to half the Amplitude of the Protection, elevated to half a right Angle, and made with the ſame Impetus. Laſtly, to make equal Randoms in a Horizontal Plane, there is required a leſs Impetus in the Projection whoſe Elevation is half a right Angle: for if it is not leſs than the Impetus of another Projection, made in another Direction; the Amplitude of the Projection whoſe Elevation is half a right Angle, will be greater than the Amplitude of that other Projection.

Cor. 2. BECAUSE AK touches the Circle, it will be [by 32 *El.* 3.] the Angle ABE=EAK the Angle of Elevation; and conſequently, the Angle AGE is doube EAK. Wherefore GA, half the Impetus, being made Radius, EF the fourth part of the Amplitude, will be the Sine of double the Angle of Elevation; and AF, the Height of the Projection, will be the verſed Sine of the Arch AE, or of double the Angle of Elevation; and FB, the Sublimity of the Parabola, will be the verſed Sine of the Arch BE, or of the Complement of double the Angle of Elevation to two right Angles.

PROBL. IX.

THE *Amplitude* AK, *and the Angle of Direction* CAK, *being given*; *to find the Impetus of the Projection, and the Height* AI.

TAKE AD the fourth part of the Amplitude; and erect the Perpendiculars DC, AB; and make the Angle ACB a right one: I ſay, AB will be the



Impetus of the Projection, and DC will be its Height. For becauſe the Angle ACB is a right one, the Semi-Circle deſcribed on the Diameter AB, will paſs through C; whence [by Corol. 1. of the preceding Problem] the Projection whoſe Direction is A C, Impetus A B, will deſcribe in its Motion the Parabola A M K, whoſe Height is D C or A I, and the fourth part of its Amplitude is A D; wherefore converſly the Projectile whoſe Direction is A C, fourth Part of its Amplitude A D, will have the Impetus A B, and Height D C. *Q. E. D.*

Cor. 1. HENCE from any Horizontal Random of any Engine being given, and made at a given Elevation; may be found the Height to which a Ball will riſe, being ſhot directly upwards: to wit, the Impetus of the Engine, which in great Ordnance exceeds any perpendicular Height to which any Man can come at. But the Impetus being given, the Amplitude and the Height of a Random made from any Elevation, will be given: whence may be diſcovered, whether a Mark, whoſe Diſtance is known, can be hit by a given Engine.

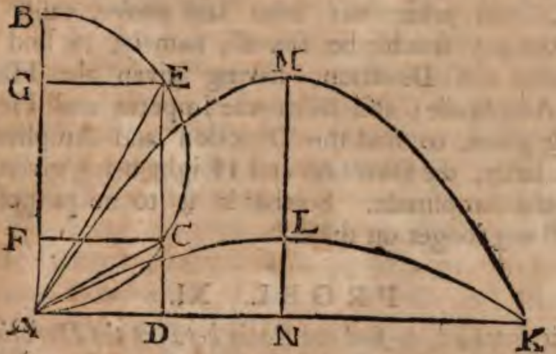
Cor. 2. IF A D, the fourth part of the Amplitude, is made Radius, the Height D C will be the Tangent of the Angle of Elevation. To ſtrike a Mark at any given Horizontal Diſtance, it is beſt to keep always the ſame Angle of Direction, namely, half a right one, and to increaſe or diminiſh the Impetus, till the Mark is hit. For the Engine elevated to this Angle, requires the leaſt Impetus to ſtrike the Mark: ſo that in making theſe Randoms, much Gun-powder is ſaved. Beſides, about this Elevation the Randoms are moſt certain; for an Error of one or two Degrees ſcarce produces a ſenſible Error in the Projection.

PROBL. X.

THE *Impetus and Amplitude being given, to find the Direction and Height of the Random.*

LET A B be the Impetus, and A D the fourth part of the given Amplitude; upon the Diameter A B

A B deſcribe a Semi-circle A C E B, and erect the Normal D C E, cutting the Semi-circle in the Points C and E: I ſay, both the Directions, whether A C



or A E, will deſcribe a Parabola, whoſe Amplitude will be A K quadruple of A D. For the Projections made with the Impetus A B, in the Directions A C or A E, have the Amplitude A K quadruple of F C or G E, [by Probl. 8.] but the Height may be either A F or A G, as is manifeſt. But if the Normal D C meets the Circle in one Point only, that is, touches it; there will be but one Parabola deſcribed, and that by a Projection whoſe Elevation is half a right Angle: and the propoſed Amplitude will be the greateſt poſſible with that Impetus. If the Perpendicular D C does not meet the Semi-circle; the Problem will be impoſſible.

Cor. IF the Impetus of any Engine is known, (as it may be found by Corol. 1. of the preceding Probl. from any Horizontal Random) by the help of this Problem, ſuch an Elevation may be given to the Engine, that the Mark placed at any given Horizontal Diſtance may be ſtruck; and of two Directions fit for the purpoſe, equally remote from the Direction whoſe Elevation is half a right Angle, the more proper may be made choice of.

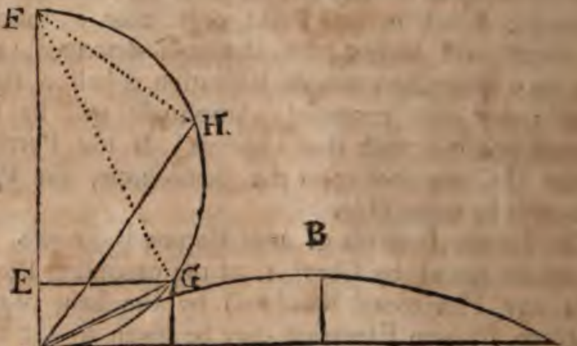
SCHOLIUM.

THE Converſe of the three preceding Problems, may, from what has been ſaid above, eaſily and without any trouble be ſolved; namely, to find the Impetus and Direction, having given the Height and Amplitude: alſo from the Impetus and Height being given, to find the Direction and Amplitude: And laſtly, the Direction and Height being given, to find the Amplitude. So that it is to no purpoſe to dwell any longer on theſe.

PROBL. XI.

IT *is propoſed, to find the Ratio betwixt the Duration of a Projection made perpendicularly upwards, and of any other, whoſe Impetus is the ſame.*

LET AF be the Impetus of the Projectile, or the Projection made upwards, and ABC the Projection from any other Elevation AG; about the Diameter



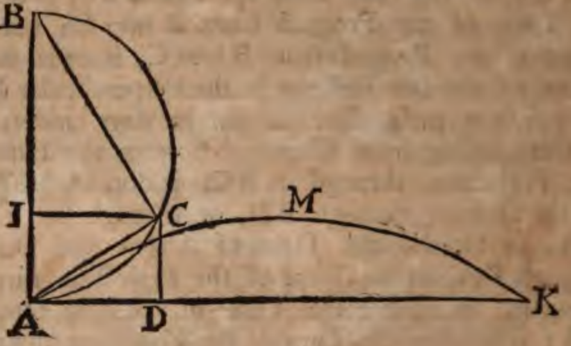
AF deſcribe a Semi-circle, cutting the Direction AG in G: I ſay, the Duration of the Projection directly upwards, or the Time of Aſcent through AF, A F, and of Deſcent through the ſame, is to the Duration of the Projection in the Parabola A B C, as A F to A G. The Time of the Progreſs from A into B, is equal to the Time of the Progreſs from B into C; ſo that the Time through A B C, is double the Time of the Progreſs from B into C; but the Time of the Progreſs from B into C, is equal to the Time of the free Deſcent in the Perpendicular B D, becauſe the progreſſive Motion no ways hinders the Deſcent ariſing from Gravity. So that the Time of the Projection through A B C, is double the Time of the Deſcent through B D, or through its Equal E A; ſo likewiſe the Time of Aſcent and Deſcent through F A, or the Time of the Projection directly upwards, is double the Time of Deſcent through F A; wherefore the Time of the Projection upwards, will be to the Time of the Projection in the Parabola A B C, as the Time of Deſcent through F A, to the Time of Deſcent through E A; that is, in a ſubduplicate Ratio of F A to E A; or, by reaſon F A, A G, E A, are continual Proportionals, as F A to A G. *Q. E. D.*

Cor. THE Durations of Projections made with an equal Impetus in different Directions A G, A H, are in the Ratio of the Chords A G, A H. And if A F is made Radius, A G will be the Sine of the Angle A F G, which is equal to the Angle of Elevation of the Engine: ſo that the Time of the Projection directly upwards, is to the Time of Projection in a Parabola, as Radius to the Sine of the Angle of Direction.

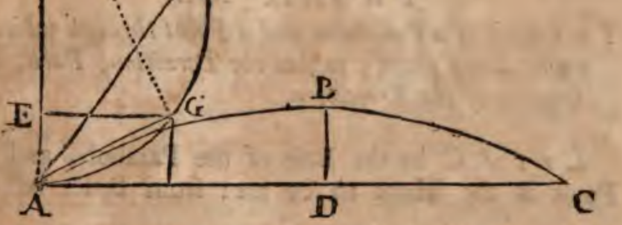
SCHOLIUM.

ALL the Problems about the Projections of heavy Bodies made in a Horizontal Plane, are eaſily reſolved by the means of a Table of Sines and Tangents.

LET AK be the Horizontal Amplitude of any large Piece of Ordnance, elevated to the given Angle CAK, there is required the Height of the Projection, and the Impetus of the Engine. In the Triangle ADC let it be as the Radius to the Tangent of the Angle of Elevation; ſo is AD, the fourth part of the given Amplitude, to the Height DC. Alſo make as the Sine of the Angle of Elevation to Radius, ſo the Height found DC to AC, which therefore will be given; and in the rectangle Triangle BCA, make as the Sine of the Angle ABC (which is equal to the Angle of Elevation) to Radius, ſo AC to AB the Impetus, which conſequently is known. But the Impetus being given, the Time of the perpendicular Projection will be likewiſe given. But the Time of the perpendicular Projection is to the Time of the Projection in AC, as AB to AC, or as Radius to the Sine of the Angle of Elevation; and conſequently the Time of Projection in AC may be known from a Table of Sines. Hence alſo from the Time of any Projection being given, made according to a given Elevation, will be given the Time of any other Projection, made with the ſame Impetus. For as the Sine of Elevation of a Projection,



jection, whoſe Time is known, is to the Sine of another Elevation; ſo is the Time known of one Projection, to the Time of the other Projection, which will therefore be alſo known. But from the Amplitude of one Projection being given, made in a given Direction, will be given the Amplitude of a Projection made in any other Direction. For making half the Impetus Radius, the fourth part of the Amplitude will be the Sine of double the Angle of Elevation; and conſequently the Amplitudes are as the Sines of theſe Angles. Wherefore if the Amplitude in the Direction A G is known, the Amplitude in the Direction A H will be given: for make as the Sine of double the Angle C A G, to the Sine of double the Angle H A C, ſo the Amplitude of the Projection according to A G, to the Amplitude of the Projection according to the Direction A H; and if from the Impetus and Horizontal Amplitude being given, there is ſought the correſponding Elevation, that may be eaſily diſcovered from the ſame Principle. For it is manifeſt from Cor. 2. Probl. 8. that double the Impetus is the Amplitude of the Projection whoſe Elevation is half a right Angle. But the Sines of the Elevations doubled are as the Amplitudes;



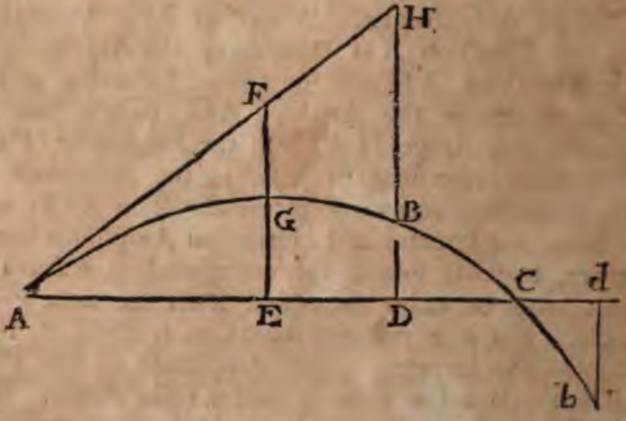
plitudes; wherefore make as double the Impetus to the given Amplitude, ſo the Sine of double half a right Angle, that is, the Sine of ninety Degrees, or Radius to another; which will be the Sine of two Arches, whereof one is the Complement of the other to a Semicircle: and theſe two Arches halved, will give two Elevations, wherewith the given Amplitude may be reached.

THE warlike Engines are not always to be ſo exploded, as the Bullet ſhall fall preciſely in the ſame Horizontal Plane; but the Mark is often higher or lower than the Ordnance: wherefore in the following Problem, a Method is delivered, whereby a Mark may be hit, either above or below the Horizon.

PROBL. XII.

THE *Baſe of a Parabola, and a Point through which it paſſes, being given; to find the Direction, Path, and Impetus of the Projection.*

LET AC be the Baſe of the Parabola, and the Point B the Mark to be hit: from B let fall the



Perpendicular B D on A C; take L a fourth Proportional to the right Lines B D, A D, D C; L will be the *Latus Rectum* of the Parabola. Biſect A C in E, and from E erect the Perpendicular E F; let E G be a third proportional to the right Lines L and A E; G will be the Vertex of the Parabola: and if E G is produced, ſo that it is $GF = GE$, and A E be drawn, F A E will be the Angle of Direction of the Engine. And the Impetus wherewith the heavy Body is to be projected, will be equal to $EG + \frac{1}{4}L$. Becauſe B D is to A D as D C to L; it will be $L \times BD = AD \times DC$; ſo that [by Cor. Theor. 48.] L is the *Latus Rectum* of the Parabola, paſſing through B, whoſe Baſe is A C. And becauſe L, A E, E G, are Proportionals, it will be $L \times EG = AE$q. ſo that G will be the Vertex of the Parabola. Therefore with the Vertex G, and *Latus Rectum* L, a Parabola being deſcribed, it will be the Path of the Projection of the heavy Body, that will ſtrike the Point B. And the Impetus of the Projection is equal to $EG + \frac{1}{4}L$: but the Angle of Elevation is F A E. *Q. E. I.*

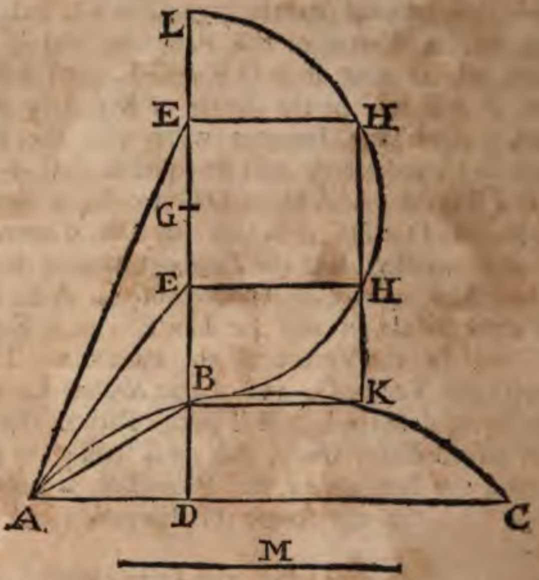
You muſt proceed after the ſame manner, if the Point *b* is below the Horizon; for if from *b* be let fall the Perpendicular *b d* on A C produced, and L is taken a fourth Proportional to *b d*, A *d*, *d* C, L will be the *Latus Rectum* of the Parabola paſſing through *b*.

Cor. A E being made Radius, E F or the Double of E G will be the Tangent of the Angle of Elevation; ſo that if it be made as A E, which is given, to E F alſo given, ſo Radius to the Tangent of the Angle F A E, the Angle of Elevation will be given.

PROBL. XIII.

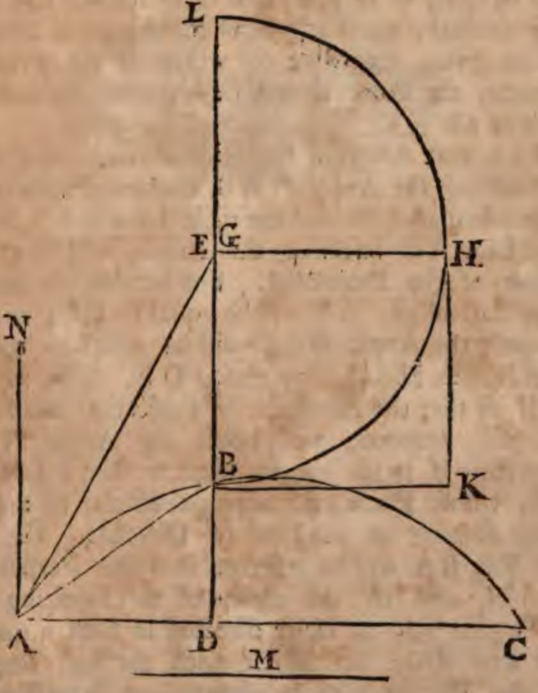
The *Impetus being given, to find the Direction according to which a heavy Body being projected, that Body may hit a given Point.*

LET M be the given Impetus, and B the Point through which the Projectile ought to paſs, whoſe Diſtance A B from the Point A is given: from B on



the Horizontal Line A C let fall the Perpendicular B D, in which produced take $DG = 2M$, and with the Center G and Interval G B deſcribe a Circle, which let the right Line $BK = AB$ touch in B. From K upon B K erect the Perpendicular K H, meeting the Circle in two Points H, H: from which on the Diameter L B let fall the Perpendiculars H E, H E, and draw the right Lines A E, A E, which will be the two Directions ſought: that is, a Projectile emitted in the Direction A E with the Impetus M, will paſs through the Point B. For it is $ADq. + BDq. = ABq. = BKq. = EHq. =$ (from the Nature of a Circle) $LE \times EB = LB \times EB - EBq. = 4M \times EB - 2DB \times EB - EBq.$ Wherefore it will be $4M \times EB = (ADq. + BDq. + 2BD \times EB + EBq. = ADq. + DEq. =)\ AEq.$ But the Parabola deſcribed by a heavy Body, projected in the Direction AE, with the Impetus M, will ſo cut the right Line DE, that it is $4M \times EB = AEq.$ (as is manifeſt from Cor. 2. Theor. 51.) Wherefore the Point B is in the ſame Parabola; and the heavy Body projected with the Impetus M, in the Direction AE will paſs through B. *Q. E. D.*

Cor. If HK meets the Circle in one Point only, that is, if it touches the Circle, there will be only



one Direction that will ſatisfy the Condition. But if it does not meet the Circle at all, the Problem will be impoſſible; that is, the Point B cannot be hit with the given Impetus. And if K H touches the Circle, that Impetus will be the leaſt of all, wherewith the given Point can be hit. And in that caſe it will be B K or $AB = BE$ or $BG = 2M - DB$; ſo that it will be $BE + BD$ or $DE = 2M$. The leaſt Impetus therefore, wherewith a given Point may be hit, will be equal to half $DE = \frac{AB + BD}{2}$; and D A being made Radius, D E will be the Tangent of the Angle E A D, that is, of the Angle of Elevation. Wherefore if it be made as A D to D E, or to $AB + BD$, ſo Radius to a fourth Proportional; the Tangent of the Angle of Direction will be given, according to which if the Projection is made, the Point B will be hit with the leaſt Impetus of all.

But that Angle of Direction is much eaſier found, by biſecting the Angle N A B, contained betwixt the Perpendicular A N and the right Line A B. For the right Line A E biſecting this Angle, will be the Direction of the Projection. For becauſe the Impetus is the leaſt of all, A B will be equal to E B; and conſequently the Angle B A E will be equal to the Angle $BEA = NAE$, (by reaſon D E, A N, are parallel) ſo that the Direction of Projection made with the leaſt Impetus, will biſect the Angle N A B. Wherefore if to the Engine there is fixed a Looking-Glaſs, whoſe Plane ſhall be perpendicular to the Axis of the Engine or the Line of Direction; the incident Ray B A will be reflected into the Perpendicular A N, and by the means of this Looking-Glaſs the Engine may be eaſily directed in ſuch a manner, as it may ſtrike the Mark with the leaſt Impetus. For the Engine is to be elevated or depreſſed, till the Image of the Point B, made on the Plane of the Looking-Glaſs, may be ſeen in the Perpendicular

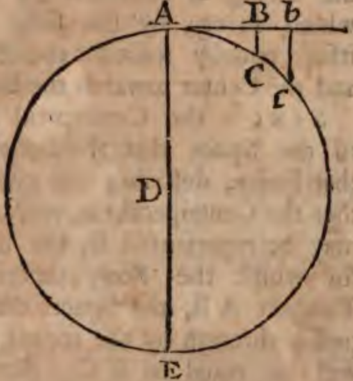
N A: for by reaſon the Angle of Incidence B A E is equal to the reflected Angle N A E, the Angle N A B will be biſected, and A E will be the Direction of the Engine, when the Point B may be hit with the leaſt Impetus.

THE

THE
THEOREMS
Of the Famous
Monſ. *HUYGENS*,
CONCERNING
The CENTRIFUGAL FORCE, and
CIRCULAR MOTION,
DEMONSTRATED.

I *FIRST communicated the Demonſtrations of the following Theorems to the Learned World; for their Author publiſhed them without any Demonſtration. But afterwards the ſame Theorems were demonſtrated by ſome* French-men, *but not in the ſame Order; and now at length the Author's own Demonſtrations are publiſhed in his Poſthumous Pieces, which though they are indeed very ſhort, yet they are more prolix than ours. But ſince theſe Theorems make no mean Part of the Science of Motion,* *we have thought fit to join their Demonſtrations to this Work; that the Commonwealth of Learning may ſee how much the Mechanical Philoſophy is capable of being improved by Geometry.*

Defin. I. A CENTRIPETAL Force is that Force, whereby any moving Body is continually drawn from its rectilinear Motion, and is perpetually ſollicited towards ſome Center. For ſince, according to a well-known Law of Nature, every Body once in Motion endeavours to proceed always uniformly in the ſame right Line, it is manifeſt no moving Body can deſcribe an Orbit in its Motion, unleſs it is detained in that Orbit by ſome Force. For example, let a moving Body revolve with a uniform Motion in the Circumference of the Circle A C E, which when it comes to A, if the Force that kept it in the Orbit is taken away, it will proceed along the Tangent A B, and run out *in infinitum*: that it therefore may be detained in the Circumference, it is neceſſary that ſome Force continually acts, and which is equivalent to the Force in A ſolliciting the Body towards D through a Space equal to B C, in the mean time whilſt the moving Body by its innate Force proceeds through the infinitely ſmall Space A B: For after this manner, by theſe Forces joined together, the moving Body will deſcribe the Line A C [by Theor. 30.] This Force, whether it is the Action of a



String holding the Body, or its Coherence with another revolving Body, or whether it ariſes from Gravity or any Attraction whatſoever, it may be called a Centripetal Force.

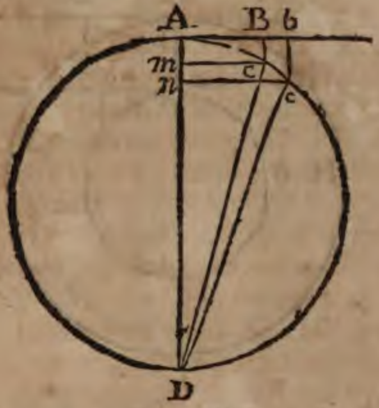
2. A CENTRIFUGAL Force is the Re-action or Reſiſtance which a moving Body exerts to prevent its being turned out of its way, and whereby it endeavours to continue its Motion in the ſame Direction: and as Re-action is always equal, and contrary to Action, ſo in like manner is the Centrifugal to the Centripetal Force. This Centrifugal Force ariſes from the *Vis Inertiæ* of Matter, and when a Body revolving in the Circumference of a Circle, is detained by the means of a String from running out; the String is ſtretched by this Centrifugal Force: which String, by the ſame Endeavour of relaxing itſelf, equally ſollicits the Body towards the Center, and the Center towards the Body.

SINCE the Centripetal Force is proportionable to the Space that the Body, by the Sollicitation of that Force, deſcribes in a given Time; it is manifeſt, that the Centripetal, as well as the Centrifugal Force, may be repreſented by the naſcent Lines B C or *b c*: for whilſt the Body deſcribes the infinitely ſmall Tangent A B, the Space that in the mean time it paſſes through by the means of the centripetal Force, will be equal to B C. But we have demonſtrated (Lect. 4.) in naſcent or infinite ſmall Lines A B, A C, that BC is infinitely leſs than A B or A C. Whence the Centripetal or Centrifugal Force will be infinitely leſs than the communicated Force A B.

LEMMA I.

IN *a Circle the evaneſcent or infinitely ſmall Subtenſes of the Angle of Contact, are in a duplicate Ratio of the conterminal Arches.*

LET the Arches be A C, A *c*, the Subtenſes perpendicular to the Tangent B C, B *c*; draw the Diameter A D, and C *m*, *c n*, Perpendiculars to the Diameter; and it will be $BC : bc :: Am : An :: Am \times AD : An \times AD$. But it is [by 8 *El.* 6.] $AD : AC :: AC : Am$, and $AD : Ac :: Ac : An$; wherefore is will be $AD \times Am = ACq$, and $AD \times An = Acq$; wherefore it is alſo $BC : bc :: ACq : ACq$. *Q. E. D.*



Cor. HENCE it is $BC = \dfrac{ACq}{DA}$.

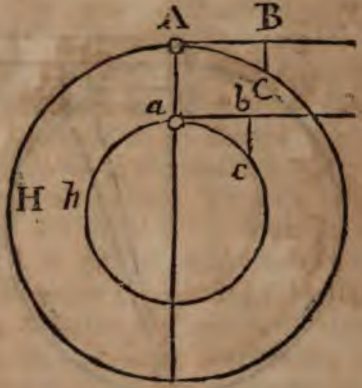
THIS *Lemma the Great Sir* Iſaac Newton *has univerſally demonſtrated in all Curves of the firſt Order.*

THEOR. I.

IF *two equal Bodies in Motion run through unequal Circumference in equal Times; the Centrifugal Force in the greater Circumference, will be to that in the leſs, as the Circumferences amongſt themſelves, or their Diameters.*

LET the Body in motion A run through the Circumference A C H, and in the ſame Time let the

Body *a* run through the Circumference *a c b*; and let A C, *a c* be the infinitely ſmall Arches deſcribed in the ſame Time. Becauſe both Circumferences are run through in an equal Time, theſe Arches will be ſimilar, and conſequently the Figure A B C will be ſimilar to the Figure *a b c*; wherefore B C : *b c* :: A C : *a c* :: Circumference A C H : to Circumference *d c b*. But it is manifeſt from the foregoing Definition, that the Centrifugal Force of the Body A is to the Centrifugal Force of the Body *a*, as B C to *b c*. Wherefore the Centrifugal Force of the Body A will be to the Centrifugal Force of the Body *a*, as the Circumference A C H to the Circumference *a c b*, or as the Diameter of that to the Diameter of this. *Q. E. D.*

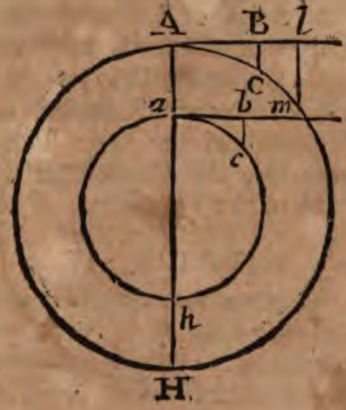


Cor. HENCE converſly, if the Centrifugal Forces are as their Diameters, the Periodical Times will be equal.

THEOR. II.

IF *two equal Bodies in Motion are carried in unequal Circumferences with an equal Celerity, their Centrifugal Forces will be in a reciprocal Ratio of the Diameters.*

Let AC, *ac*, be the leaſt Arches deſcribed in the ſame Time, which, by reaſon the Velocity in both the moving Bodies is equal, will be equal. Make the Arch A *m* ſimilar to the Arch *ac*, and draw *lm* parallel to BC; and the Centrifugal Force in the greater Circumference will be to that which is in the leſs, as the naſcent Line BC to the naſcent Line *bc*: but BC is to *bc* in a Ratio compounded of BC to *lm* and *lm* to *bc*; and from the preceding Lemma, BC is to *lm*, as AC*q* to A*mq*, and *lm* is to *bc* as A*m* to *ac* or AC. Wherefore it will be $BC : bc :: ACq : Amq + Am : ac :: ACq : Amq + Amq : Am \times ac :: ACq$ or $acq : Am \times ac :: ac : Am$; that is, as the whole Circumference *acb* to the whole Circumference ACH, or as the Diameter *ab* to the Diameter AH. *Q. E. D.*



THEOR. III.

If two equal moving Bodies are carried in equal Circumferences, but both with an equable Motion, (as we would be underſtood to mean in all) the Centrifugal Force of *the ſwifter will be to the Force of the ſlower in a duplicate Ratio of the Celerities.*

For the Centrifugal Forces are as the evaneſcent Subtenſes of the Angle of Contact, which (by what we have already demonſtrated) are in the ſame or equal Circles in a duplicate Ratio of the conterminal Arches; but the conterminal Arches, ſince they are the Spaces deſcribed in the ſame Time, are as the Velocities: wherefore the Centrifugal Forces are in a duplicate Ratio of the Velocities. *Q. E. D.*

THEOR. IV.

If *two equal Bodies in motion, carried in unequal Circumferences, have their Centrifugal Forces equal; the Time of the Revolution in the greater Circumference, will be to the Time of the Revolution in the leſs, in a ſubduplicate Ratio of the Diameters.*

Let AC, *ac*, be the leaſt Arches deſcribed in the ſame Time; (*ſee* Fig. Theor. 2.) becauſe the Centrifugal Forces are equal, it will be $BC = bc$. Let the Time wherein the Circumference ACH is deſcribed, be called T, and the Time wherein the Circumference *ach* is deſcribed, be *t*: make the Arch A*m* ſimilar to the Arch *ac*, and let us ſuppoſe any moving Body to run through the Circumference ACHA in the ſame Time, as it would run through the Circumference *acha*; and in that Caſe the Arches deſcribed in the ſame Time in both Circumferences will be A*m*, *ac*. But the Velocity of a moving Body paſſing through the Arch A*m* in any given Time, is to the Velocity of a moving Body paſſing through the Arch AC in the ſame Time, as the Arch A*m* to the Arch AC: ſo that ſince the Time wherein the ſame Circumference is run over, is always reciprocally as the Velocity, it will be $T : t :: Am : AC$, and $T^2 : t^2 :: Amq : ACq :: ml : BC :: ml : bc$; that is, by reaſon the Arch AM is ſimilar to the Arch ac, as the Diameter AH to the Diameter ab: whence it is manifeſt, that $T : t :: \sqrt{AH} : \sqrt{ab}$. *Q. E. D.*

Schol. SINCE in every Caſe the Centrifugal Force is to the Centrifugal Force as BC to bc, but it is $BC = \frac{ACq}{AH}$ and $bc = \frac{acq}{ab}$; the Centrifugal Force will be to the other Centrifugal Force as $\frac{ACq}{AH}$ to $\frac{acq}{ab}$: that is, as the Squares of the Arches deſcribed in the ſame Time applied to the Diameters of the Circles. And ſince thoſe Arches are as the Velocities, the Centrifugal Forces will be as the Squares of the Velocities applied to the Diameters of the Circles.

LEMMA II.

IF *a moving Body revolves in the Circumference of a Circle, the Space, which the moving Body proceeding forwards, and only the Centrifugal Force ariſing from that circular Motion taking place, would run over in a given Time, will be a third Proportional to the Diameter of the Circle and that Arch, which if it was carried in the Circumference of the Circle it would deſcribe in the ſame Time.*

LET AC be any Arch deſcribed in any the leaſt Particle of Time, and let n denote any Time or any Number of thoſe Particles, $n \times AC$ will be the Arch which the moving Body carried in the Circumference will deſcribe in the given Time n, and BC the Space which it would run over in the firſt Particle of that Time, the Centrifugal Force taking place. But ſince every Body in motion, by the ſame Force continued always in the ſame Direction, deſcribes Spaces in a duplicate Ratio of the Times, [by Cor. 3. Theor. 12. Lect. 11. for whatever is demonſtrated of Gravity, may be applied to any other Force acting uniformly] the Space deſcribed, the Centrifugal Force acting, in the Time n, will be $= n^2 \times BC$. But (as is manifeſt from the firſt Lemma) it is $AH : AC :: AC : BC$, and as AC to BC, ſo $n \times AC$ to $n \times BC$. Wherefore AH is to AE as $n \times AC$ to $n \times BC$; and by multiplying the Conſequents by n, AH will be to $n \times AC$ as $n \times AC$ to $n^2 \times BC$: that is, the Diameter of the Circle, the Arch deſcribed in a given Time, and the Space which, the Centrifugal Force acting, is run over in the ſame Time, are continual Proportionals. *Q. E. D.*

Cor. If the Diameter of the Circle is called D, and the Arch deſcribed by the moving Body, in any Time A, the Space which the moving Body, the Centrifugal Force acting, proceeding forwards would deſcribe in the ſame Time, will be $\frac{A^2}{D}$; for D, A, $\frac{A^2}{D}$, are continual Proportionals.

THEOR. V.

If a moving Body is carried in the Circumference of a Circle, with the Celerity which it has acquired by falling from a Height that is equal to a fourth Part of the Diameter, it will have a Centrifugal Force equal to its Gravity; that is, it will ſtretch the String, whereby it is kept to the Center, juſt as much as if it was ſuſpended by it.

Let the Diameter of the Circle be called D, and the Circumference P: and ſince, by Hypotheſis, the Velocity of the moving Body carried in the Circumference is uniform, and equal to that which it acquires by falling through $\frac{1}{4}$ D, it is certain that the moving Body carried in the Circumference will deſcribe in an equal Time an Arch equal to twice that [by Theor. 12. Lect. 11.] that is, $= \frac{1}{2}$ D; whence [by Lemm. 2.] the Space paſſed over in the mean while, by the acting of the Centrifugal Force, will be $= \frac{1}{4}$ D. For D is to $\frac{1}{2}$ D, as $\frac{1}{2}$ D to $\frac{1}{4}$ D: but by Hypotheſis the Space that the moving Body by the Force of Gravity deſcribes in the ſame Time, is alſo $\frac{1}{4}$ D. Wherefore ſince the Spaces run over by theſe two Forces in the ſame Time, are equal, the Forces themſelves will be likewiſe equal.

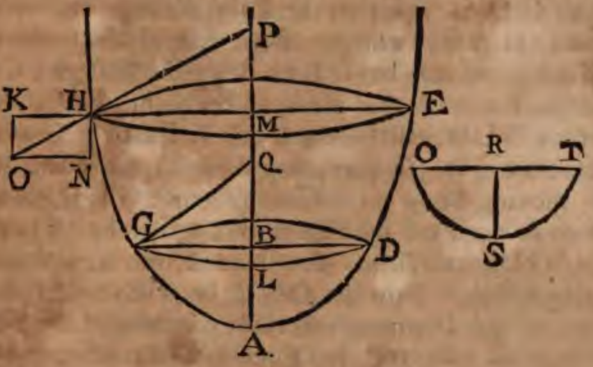
Cor. 1. HENCE converſly, if a moving Body carried in the Circumference of a Circle has a Centrifugal Force equal to its Gravity, its Velocity is that which is acquired by falling through $\frac{1}{4}$ D.

Cor. 2. HENCE the Time of a Revolution is to the Time of Deſcent through $\frac{1}{4}$ D, as P to $\frac{1}{2}$ D, or as 2 P to D. For in the Time a moving Body, with an accelerated Velocity, runs over $\frac{1}{4}$ D, it will, moved uniformly with the Velocity acquired at the laſt, run through $\frac{1}{2}$ D: and conſequently ſince the Velocities are equal, the Times will be as the Spaces run over; that is, the Time wherein the moving Body runs through the Circumference, is to the Time wherein it deſcribes $\frac{1}{2}$ D, as P to $\frac{1}{2}$ D, or as 2 P to D; but the Time wherein $\frac{1}{2}$ D is deſcribed, is = Time of the Fall through $\frac{1}{4}$ D: whence the Time of the Revolution will be to the Time of the perpendicular Fall through $\frac{1}{4}$ D, as 2 P to D.

THEOR. VI.

IN *the concave Superficies of a Parabolick Conoid, which has its Axis erected perpendicularly, all the Revolutions of a moving Body running through the Circumferences parallel to the Horizon, whether they are ſmall or great,* *are performed in equal Times; each of which Times are equal to two Oſcillations of a Pendulum, whoſe Length is half the* Latus Rectum *of the generating Parabola.*

LET HGADE be a Parabolick Conoid, whoſe Axis erected perpendicularly is A P; G D, H E, the Diameters of Circles, whoſe Circumferences parallel to the Horizon the moving Body runs through; which therefore will be ſollicited by three Powers mutually equivalent to each other, acting in three different Directions: whereof the firſt is the Force of Gravity impelling the moving Body in the right Line H N perpendicular to the Plane of the Horizon; the ſecond is the Centrifugal Force ariſing from the circular Motion, ſolliciting the moving Body from H towards K; but the Place of the third Power is ſupplied by the Reſiſtance or contrary Endeavour of the Parabolick Superficies, according to the Line H P acting perpendicularly to it; for Re-action is always equal to Action, and towards the contrary Part. Whence ſince the Superficies is preſſed perpendicularly by the moving Body, it re-acts on the Body according to the Direction H P, and that contrary Endeavour is equivalent to the Power ſolliciting the moving Body according to the Direction H P. Wherefore ſince the moving Body is ſuſtained by theſe three Powers, they muſt be neceſſarily *in æquilibrio*, that is, any two deſtroy the Effect of the other. Whence O N being drawn parallel to H K meeting H N in N, if O H repreſents the Re-action of the Parabolick Superficies, the right Line O N will expound the Centrifugal Force, and H N the Force of Gravity of the moving Body: But by reaſon the Triangles HO N, H M P, are equi-angular, O N is to H N as HM to MP; that is, the Centrifugal



fugal Force of the moving Body deſcribing the Circumference of the Circle H M E, will be to the Force of its Gravity as H M the Radius of the Circle, to M P the Subnormal. In like manner, in any other Circumference G L D, in the Superficies of the Conoid, the Centrifugal Force of the moving Body deſcribing it, is to the Force of Gravity as G B the Radius, to B Q the Subnormal. Moreover, becauſe the Centrifugal Force of the moving Body running over the Circumference H M E, is to the Force of Gravity as H M to M P; and the Force of Gravity of the ſame moving Body, is to its Centrifugal Force, when it runs over the Circumference G L D, as B Q to B G, or (from the Nature of the Parabola) as M P to B G: by Equality, the Centrifugal Force of the moving Body running over the Circumference H M E, will be to its Centrifugal Force when it runs over the Circumference G L D, as H M to B G; that is, the Centrifugal Forces are as the Semi-diameters or Diameters of the Circles, whence [by Cor. Theor. 1.] the Periodical Times are equal. Which was the firſt thing to be demonſtrated.

Let the Circle GLD be ſo taken, that its Diameter GD be equal to the *Latus Rectum* of the Parabola HAE; whence from the Nature of the Parabola, it will be GB = BQ: So that the Centrifugal Force of the moving Body in the Circumference GLD will be equal to the Force of Gravity. Therefore [by the preceding Corol.] the Velocity of the moving Body in the Circumference GLD, is that which is acquired by falling through a Space equal to $\frac{1}{4}$ GD, or (from the Nature of a Parabola) through BA. Now let OST be a Cycloid, whoſe Axis or the Diameter of the generating Circle SR is equal to AB, and the Time of Deſcent through the Cycloid OS will be to the Time of the perpendicular Fall through the Axis RS, or through BA, as $\frac{1}{2}$ P to D, by Theor. 46. Lect. 15. But [by the preceding Corol.] the Time of Deſcent through AB is to the Time of the Revolution in the Circumference GLD as D to 2 P; wherefore, by Equality, the Time of Deſcent through the Cycloid OS, is to the Time of a Revolution in the Circumference GLD as $\frac{1}{2}$ P to 2 P, or as 1 to 4: whence the Time of four Deſcents in a Cycloid, or the Time of two Oſcillations in a Cycloid, is equal to the Time of a Revolution in the Circumference GLD. But the Time of two Oſcillations in a Cycloid, is equal to the Time of two the leaſt Oſcillations in a Circle, which is of the ſame curvity with the Cycloid at the Vertex S; by reaſon a Portion of ſuch a Circle and a Portion of the Cycloid nearly coincide at the Vertex S; and conſequently in Phyſical Matters perform the ſame Effect, as is now well known. But the Radius of a Circle of the ſame degree of curvity with the Cycloid at the Vertex S, is equal to twice RS, or twice AB, (as eaſily follows from Corol. Theor. 46. Lect. 15.) So that the Length of a Pendulum oſcillating in that Circle, is equal to twice A B, or to half the *Latus Rectum* of the generating Parabola. Whence the Time of two the leaſt Oſcillations of a Pendulum, whoſe Length is half the *Latus Rectum*, is equal to the Time of two Oſcillations in the Cycloid O S T, or to the Time of a Revolution in the Circumference G L D, or in the Circumference H M E. *Q. E. D.*

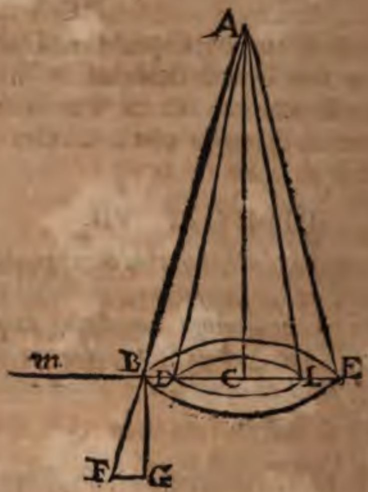
Cor. HENCE if a moving Body is carried in the Circumference of a Circle with the Celerity that is acquired by falling through $\frac{1}{4}$ of the Diameter, the Time of a Revolution will be equal to the Time of two the leaſt Oſcillations of a Pendulum, whoſe Length is the Semi-diameter of the Circle.

THEOR. VII.

IF *two Bodies, ſuſpended by unequal Strings, do ſo revolve, that they run over Circumferences parallel to the Horizon, the Top of the Strings remaining fixed, but the Heights of the Cones, whoſe Superficies are deſcribed by this Motion, ſhall be equal; the Times alſo of the Revolutions will be equal.*

LET ABE be the Cone, whoſe Superficies the String A B deſcribes; alſo A D L the Cone whoſe Superficies is deſcribed by the String A D: and let C be the Center of the Baſe of both Cones, and A C their common Height. Now let the moving Body B be conſider'd as drawn by three Powers mutually equivalent to each other: whereof one, which is the Force of Gravity, draws the moving Body in the right Line BG perpendicular to the Plane of the Horizon, the other acting according to the Direction B *m*, is the Centrifugal Force whereby the moving Body endeavours to recede from the

Center C of its Orbit; but the third, which is equivalent to and reſiſts the other two, is the contrary Endeavour of the String acting according to the Direction A B; for the Tenſion of a String is inſtead of a contrary Power, and in this caſe performs the ſame effect. If therefore B F repreſents the Action of the String, the Centrifugal Force of the moving



Body, and its Force of Gravity, will be expounded by the right Lines F G and B G, [by Theor. 28. Lect. 14.] that is, the Centrifugal Force of the moving Body B, will be to the Force of Gravity as F G to B G, or (by reaſon the Triangles F B G, A B C are equi-angular) as B C to C A. After the ſame manner, the Force of Gravity will be to the Centrifugal Force of the moving Body D, as A C to D C: wherefore, by Equality, the Centrifugal Force of the moving Body B, will be to the Centrifugal Force of the moving Body D, as B C to D C; that is, the Centrifugal Forces are as the Semi-diameters of the Circles, whoſe Circumferences the moving Bodies deſcribe, and conſequently [by Cor. Theor. I.] the Times of the Revolutions are equal. *Q. E. D.*

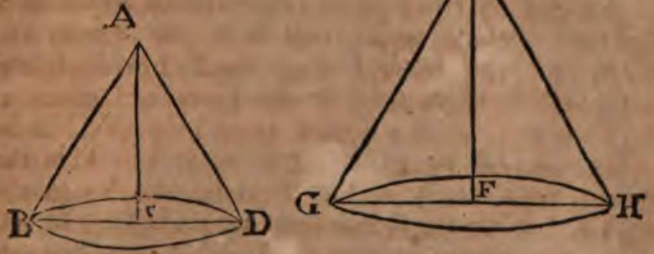
Cor. HENCE the Centrifugal Force is to the Force of Gravity as the Semi-diameter of the Baſe of the Cone to the Height of the Cone.

Note, BY the Force of Gravity and the Centrifugal Force in this Demonſtration, we underſtand the accelerating Forces of the moving Bodies, unleſs thoſe Bodies are ſuppoſed equal; in which caſe may be alſo taken the abſolute Forces.

THEOR. VIII.

I F *two Bodies, as before, revolve with a Conical Motion, ſuſpended by equal or unequal Strings, and the Heights of the Cones are unequal, the Times of the Revolutions will be in a ſubduplicate Ratio of their Heights.*

LET B and G be the two moving Bodies, and let firſt of all the Cones A B D, E G H, whoſe Super-

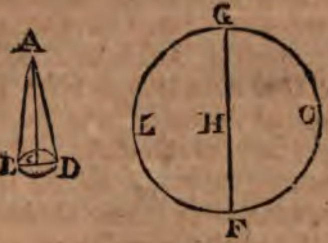


ficies are deſcribed by the Strings, be ſimilar; [by Corol. Theor. 7.] the Centrifugal Force of the moving Body B will be to the Force of Gravity as B C to A C; and the Centrifugal Force of the moving Body G will be to the ſame Force of Gravity as G F to F E. But by reaſon the Triangles A B C, G E F, are equi-angular, B C is to A C as G F to F E: wherefore the Centrifugal Force of the moving Body B will be to the Force of Gravity, as the Centrifugal Force of the moving Body G to the ſame Force of Gravity, and conſequently theſe Centrifugal Forces will be equal. Therefore, [by Theor. 4.] the Times of the Revolutions of the moving Bodies will be in a ſubduplicate Ratio of the Semi-diameters; that is, by reaſon the Triangles A B C, E G F, are equi-angular, in a ſubduplicate Ratio of the Heights A C and E F. But whatever the Cones are, which the Strings deſcribe, ſo that their Heights remain unaltered, the Times of the Revolutions will remain unchanged; wherefore in every caſe the Truth of this Theorem is manifeſt. *Q. E. D.*

THEOR. IX.

If a Pendulum carried in a Conical Motion makes very ſmall Revolutions; the Times of each of them have to the Time of the perpendicular Fall from twice the Height of the Pendulum, the Ratio, which the Circumference of a Circle has to its Diameter: and conſequently, they are equal to the Time of two very ſmall lateral Oſcillations of the ſame Pendulum.

LET A D B be a Cone, whoſe Superficies is deſcribed by a String, and let its Height be A c nearly = A B, becauſe the Revolutions are exceeding ſmall. With the Semi-diameter G H = A c deſcribe the Circle



Circle G L F O, and let a Body be ſuppoſed to revolve in its Circumference, with the Celerity that is acquired by falling through $\frac{1}{4}$ of its Diameter or $\frac{1}{4}$ D. [By Theor. 5.] its Centrifugal Force will be equal to its Gravity; but the Centrifugal Force of the moving Body B is to the Force of Gravity, and conſequently to the Centrifugal Force of the moving Body carried in the Circumference G L F, as B *c* to A *c* or G H: wherefore the moving Bodies B and G, ſince their Centrifugal Forces are as the *Radii*, will have equal Times of Revolutions, [by Cor. Theor. 1.] But the Time of Deſcent through G F or D, is to the Time of Deſcent through $\frac{1}{4}$ D as D to $\frac{1}{2}$ D, [by Cor. Theor. 12 Lect. 11.] and the Time of Deſcent through $\frac{1}{4}$ D is to the Time of a Revolution in the Circumference G L G, as $\frac{1}{2}$ D to P. Wherefore, by Equality, the Time of Deſcent through D will be to the Time of a Revolution in the Circumference G L F, or to the Time of a Revolution of the Pendulum A B *c* D as D to P. The latter part of this Theorem is manifeſt from the Corollary of Theor. 6.

Cor. HENCE ſince the Time of the perpendicular Fall is in a ſubduplicate Ratio of the Space paſſed over by a falling heavy Body, the Time of Deſcent from the Height of a Pendulum will be to the Time of an exceeding ſmall Revolution as $D \times \sqrt{\frac{1}{2}}$ to P.

THEOR. X.

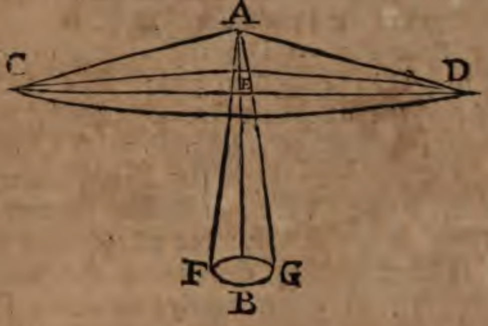
If a moving Body is carried in a Circumference, and performs each Revolution in the Time wherein a Pendulum, having its Length equal to the Semi-diameter of that Circumference, ſhall finiſh a very ſmall Revolution in a Conical Motion, or twice an exceeding ſmall lateral Oſcillation; that moving Body will have a Centrifugal Force equal to its Gravity.

Because the moving Bodies B, G, (by Hypotheſis) perform their Revolutions in equal Times, [ſee Fig. of the preceding Theor.] the Centrifugal Force of the moving Body B will be to the Centrifugal Force of the moving Body G, as B C to G H, or B C to A C; but it is as B C to A C, ſo the Centrifugal Force of the moving Body B to the Force of Gravity, [by Cor. Theor. 7.] wherefore [by 9 *El.* 5.] the Centrifugal Force of the moving Body G will be equal to the Force of Gravity. *Q. E. D.*

THEOR. XI.

The Times of the Revolutions of any Pendulum carried with a Conical Motion, will be equal to the Time of the perpendicular Fall from a Height equal to the String of the Pendulum; when the Angle of Inclination of the String to the Plane of the Horizon will be nearly 2 *degr.* 54 *min. But exactly, if the Sine of that Angle ſhall be to Radius as a Square inſcribed in a Circle to the Square of its Circumference.*

Let the Pendulum, whoſe String deſcribes the Conical Superficies C A D, be ſuch, as the Sine of



the Angle ACE ſhall be to Radius (that is, AE to AC) as $\frac{1}{2}D^2$ to P^2. Let alſo AFG be the Superficies of a Cone, which the String of the Pendulum deſcribes carried with an exceeding ſmall Motion, whoſe Height therefore is $AB = AF = AC$. The Time of a Revolution of a moving Body F, will be [by Theor. 8.] to the Time of a Revolution of the moving Body C, in a ſubduplicate Ratio of AB, or AC to AE: but it is as AC to AE, ſo (by Hypotheſis) P^2 to $\frac{1}{2}D^2$; wherefore the Time of a Revolution of the moving Body F, will be to the Time of a Revolution of the moving Body C, in a ſubduplicate Ratio of P^2 to $\frac{1}{2}D^2$, that is, in the Ratio of P to $D \times \surd\frac{1}{2}$. But it is as P to $D \times \surd\frac{1}{2}$, ſo [by Cor. Theor. 9.] the Time of the leaſt Revolution, that is, the Time of a Revolution of the Body F, to the perpendicular Fall from the Height of the Pendulum: wherefore the Time of the Revolution of the moving Body F, has the ſame Proportion to the Time of the Revolution of the moving Body C, as it has to the Time of the perpendicular Fall from a Height equal to the Length of the Pendulum; and

coin

conſequently [by 9 *El.* 4.] the Time of the Revolution of the moving Body C, will be equal to the Time of the perpendicular Fall from a Height equal to the Length of the Pendulum. *Q. E. D.*

BUT ſince P is to D nearly as 314 to 100, P^2 will be to $\frac{1}{2}D^2$ as 98596 to 5000. But A C is to A E, from the foregoing Demonſtration, as P^2 to $\frac{1}{2}D^2$; wherefore 98596 is to 5000 as A C to A E; and as A C to A E, ſo is [by Trigonometry] the Sine of the Angle A E C, or Radius 100000 to the Sine of the Angle A C E: but 98596 is to 5000 as 100000 to 5070, which therefore is the Sine of the Angle A C E, to which nearly anſwer 2 Degrees 54 Minutes.

THEOR. XII.

IF *two Pendulums equal in Weight, but having their Strings unequal, do revolve in a Conical Motion, and the Heights of the Cones are equal, the Forces wherewith they ſtretch their Strings, will be in the ſame Ratio, as is that of the Length of the Strings.*

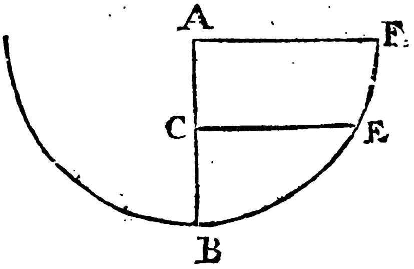
THIS is manifeſt from Theor. 7. For the Force of Gravity in both the Cones, is to the Tenſion of the String, as the Height of the Cone to the Length of the String; and ſince the Height of the Cones is the ſame, it is evident that the Tenſions of the Strings are proportionable to their Lengths. *Q. E. D.*

THEOR. XIII.

IF *a ſimple Pendulum is moved with the greateſt lateral Oſcillation, that is, if it deſcends through the whole Quadrant of a Circle, when it ſhall arrive at the loweſt Point of the Circumference, it will draw its*

String with triple a greater Force, than if it was only ſimply ſuſpended by it.

LET AB be a Pendulum moved through the Quadrant FB, biſect AB in C, through which draw CE perpendicular to AB, and meeting the Circumference in E. If the Pendulum deſcended only through the Arch EB, it would acquire in the Point B the ſame Velocity, as if it had deſcended through CB = $\frac{1}{4}$ of the Diameter [by Corol. 1. Prop. 38. of the 15th Lect.] ſo that [by Theor. 5.] it will have in the Point B a Centrifugal Force equal to its Gravity: and conſequently the Gravity and the Centrifugal Force joined together, will draw the String with twice a greater Force than it would be drawn by the Gravity alone. But if the Pendulum is raiſed to F, after its Deſcent to B, it will acquire the ſame Velocity, as if it had fallen through AB. But AB is to BC in a duplicate Ratio of the Velocity acquired in the Deſcent through AB, to the Velocity acquired in the Deſcent through BC; wherefore alſo AB will be to BC [by Theor. 3.] as the Centrifugal Force of the moving Body in the Point B after the Deſcent through FB, to the Centrifugal Force in the Point B, after the Deſcent only through EB. So that



the Centrifugal Force of the moving Body after the Deſcent through F B, will be double the Centrifugal Force after the Deſcent through E B; that is, the Centrifugal Force in the Point B, after the Fall through F B, will be double the Force of Gravity. Wherefore the String, by the Centrifugal Force and the Force of Gravity acting together, and according to the ſame Direction, is drawn by triple a greater Force, than if it was ſtretched by the Force of Gravity alone. *Q. E. D.*

FINIS.

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